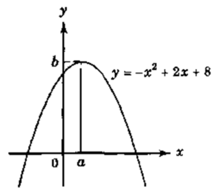
# Supplementary Exercise on Maximum and Minimum of Quadratic Function

Created by Mr. Francis Hung on 20210830 Last updated: 2021-08-30

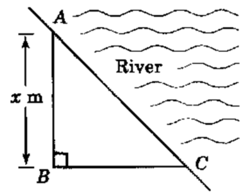
1. The figure shows the graph of the quadratic function $y = -x^2 + 2x + 8$. When $x = a$, $y$ has a maximum value of $b$. Find the values of $a$ and $b$.




2. A rectangle $ABCD$ has a perimeter of 20 cm. If $AB = x$ cm,
   (a) express the length of $BC$ in terms of $x$ ,
   (b) find the value of $x$ so that the rectangle has a maximum area.

3. The graph of $y = ax^2 + bx + c$ passes through (–1, 33) and cuts the $x$-axis at the origin and (10, 0). Find
   (a) the values of $a$, $b$ and $c$,
   (b) the minimum value of $ax^2 + bx + c$.

4. A triangular region $ABC$ is formed by two perpendicular fences ($AB$ and $BC$) and the straight shore $AC$ of a river. The two fences have a total length of 3 m and $AB = x$ m.
   (a) Express the area of the triangular region in terms of $x$.
   (b) Find the maximum area of the triangular region and the corresponding value of $x$.

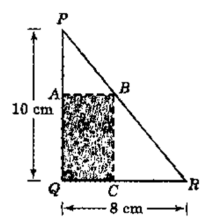



5. An object is shot vertically upwards. Its height above the ground is $h$ metres after $t$ seconds where $h = 1 + 20t - 5t^2$. (*Give the answers correct to 1 decimal place in this question.*)
   (a) Find the maximum height reached by the object and the corresponding value of $t$.
   (b) Find the values of $t$ when the object's height is half the maximum height.

6. In the graph of the quadratic function $y = ax^2 - 6ax + c$, the $x$-intercepts are 0 and $k$ whereas the highest point is (3,18).
   (a) Find the values of $c$, $k$ and $a$.
   (b) Does the graph open upwards or downwards?
   (c) Hence draw a rough diagram representing the graph of the quadratic function $y = ax^2 - 6ax + c$ .

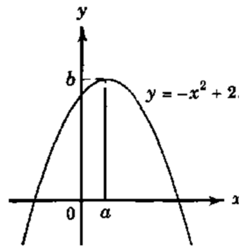
7. A mobile telephone company handles 4800 calls every day. At present each user is charged \$3 for a call. The company expects in the future that for every \$0.1 increase in the service charge of a call, the company lose 120 calls per day.
   (a) If the service charge for a call is increased by \$$x$ in the future, find in terms of $x$,
      (i) the new service charge for a call,
      (ii) the new number of calls per day expected by the company,
      (iii) the expected daily income of the company obtained from the service charge.
   (b) In order to have a maximum daily income, how much should the company charge for each call?
   (c) What is the maximum daily income in (*b*)?

8. The figure shows a sheet of cardboard $PQR$ in the form of a right-angled triangle where $PQ$ = 10 cm and $QR$ = 8 cm. A rectangle $ABCQ$ is then cut away from the cardboard as shown.
   (a) If $AB = x$ cm, express the area of $ABCQ$ in terms of $x$.
   (b) Hence determine the maximum area of $ABCQ$ and its corresponding dimensions.



1. $a = 1$, $b = 9$
2. $BC = (10 - x)$ cm, $x = 5$
3. $a = 3$, $b = -30$, $c = 0$
   Minimum = –75
4. (a) $\frac{1}{2}x(3-x)\ \text{m}^2$
   (b) maximum area $= \frac{9}{8}\ \text{m}^2$, $x = \frac{3}{2}$
5. (a) Maximum height = 21 m, $t = 2$
   (b) 3.4, 0.6
6. (a) $c = 0$, $k = 6$, $a = -2$
   (b) open downwards
7. (a) (i) $\$(3 + x)$
   (ii) $4800 - 1200x$
   (iii) $\$(3 + x)(4800 - 1200x)$
   (b) \$3.5
   (c) \$14700
8. (a) $\left(-\frac{5}{4}x^2 + 10x\right)\text{cm}^2$
   (b) 20 cm$^2$, 4 cm × 5 cm.

1. The figure shows the graph of the quadratic function $y = -x^2 + 2x + 8$. When $x = a$, $y$ has a maximum value of $b$. Find the values of $a$ and $b$.




$-y = x^2 - 2x - 8$

$\quad = x^2 - 2x + 1 - 9$

$\quad = (x - 1)^2 - 9$

$y = -(x - 1)^2 + 9$

$a = 1$, $b = 9$

2. A rectangle $ABCD$ has a perimeter of 20 cm. If $AB = x$ cm,

(a) express the length of $BC$ in terms of $x$ ,

(b) find the value of $x$ so that the rectangle has a maximum area .

(a) $BC = (10 - x)$ cm

(b) Area $= (10 - x)x$ cm$^2$

$\quad = (10x - x^2)$ cm$^2$

$\quad = [25 - (25 - 10 + x^2)]$ cm$^2$

$\quad = [25 - (5 - x)^2]$ cm$^2$

When $x = 5$, the area is a maximum.

3. The graph of $y = ax^2 + bx + c$ passes through $(-1, 33)$ and cuts the $x$-axis at the origin and $(10, 0)$. Find

(a) the values of $a$, $b$ and $c$,

(b) the minimum value of $ax^2 + bx + c$.

(a) $\because$ It passes through the $x$-axis at the origin and (10,0).

$\therefore$ The roots of the quadratic equation $ax^2 + bx + c = 0$ are 0 and 10.

$y = a\,x\,(x - 10)$

$\because$ It passes through $(-1,33)$

$33 = a\times(-1)\times(-1 - 10)$

$\Rightarrow a = 3$

$\therefore y = 3\,x\,(x - 10) = 3x^2 - 30x$

$a = 3$, $b = -30$, $c = 0$

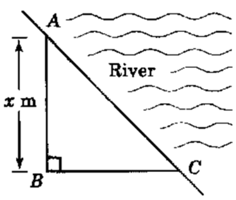
(b) $ax^2 + bx + c = 3x^2 - 30x$

$\quad = 3(x^2 - 10x + 25) - 75$

$\quad = 3(x - 5)^2 - 75$

Minimum value of $ax^2 + bx + c$ is $-75$.

4. A triangular region $ABC$ is formed by two perpendicular fences ($AB$ and $BC$) and the straight shore $AC$ of a river. The two fences have a total length of 3 m and $AB = x$ m.




(a) Express the area of the triangular region in terms of $x$.

(b) Find the maximum area of the triangular region and the corresponding value of $x$.

(a) $AB = x$ m, $BC = (3 - x)$ m.

Area $= \dfrac{1}{2}x(3 - x)\,\text{m}^2$

(b) 2 Area = $(3x - x^2)$ cm$^2$

$= \left[\frac{9}{4} - \left(\frac{9}{4} - 3x + x^2\right)\right]$ m$^2$

$= \left[\frac{9}{4} - \left(\frac{3}{2} - x\right)^2\right]$ m$^2$

Area $= \left[\frac{9}{8} - \frac{1}{2}\left(\frac{3}{2} - x\right)^2\right]$ m$^2$

When $x = \frac{3}{2}$, maximum area $= \frac{9}{8}$ m$^2$

5. An object is shot vertically upwards. Its height above the ground is $h$ metres after $t$ seconds where $h = 1 + 20t - 5t^2$.

(a) Find the maximum height reached by the object and the corresponding value of $t$.

(b) Find the values of $t$ when the object's height is half the maximum height.
(*Give the answers correct to 1 decimal place.*)

(a) $h = 1 + 20t - 5t^2$

$= -5\,(t^2 - 4t) + 1$

$= -5(t^2 - 4t + 4) + 20 + 1$

$= -5(t - 2)^2 + 21$

When $t = 2$, maximum height = 21 m.

(b) When the height $= \frac{1}{2} \times 21\,\text{m} = 10.5$ m,

$1 + 20t - 5t^2 = 10.5$

$5t^2 - 20t + 9.5 = 0$

$10t^2 - 40t + 19 = 0$

$t = \frac{20 \pm \sqrt{210}}{10} = 3.4,\ 0.6$ (correct to the nearest 1 decimal place.)

6. In the graph of the quadratic function $y = ax^2 - 6ax + c$, the $x$-intercepts are 0 and $k$ whereas the highest point is (3,18) .

(a) Find the values of $c$, $k$ and $a$.

(b) Does the graph open upwards or downwards?

(c) Hence draw a rough diagram representing the graph of the quadratic function $y = ax^2 - 6ax + c$.

(a) $y = a\,(x - 3)^2 + 18$ ($\because$ the highest point is (3,18))

It passes through (0,0)

$0 = a\,(-3)^2 + 18$,

$a = -2$.

$y = -2\,(x - 3)^2 + 18$

$y = -2(x^2 - 6x + 9) + 18$

$y = -2x^2 + 12x$, which is identical to $y = ax^2 - 6ax + c$

$\therefore c = 0$.

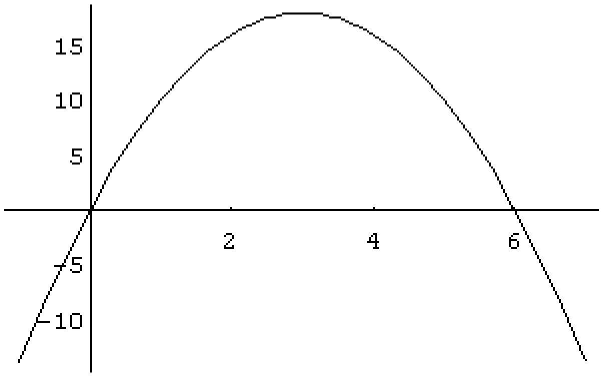
$y = -2x(x - 6)$

$\because$ the other $x$ intercept is 6, $\therefore k = 6$.

In conclusion, $a = -2$, $c = 0$, $k = 6$.

(b) $\because a = -2 < 0$, $\therefore$ the graph opens downwards.

(c)




7. A mobile telephone company handles 4800 calls every day. At present each user is charged \$3 for a call. The company expects in the future that for every \$0.1 increase in the service charge of a call, the company lose 120 calls per day.

(a) If the service charge for a call is increased by \$$x$ in the future, find in terms of $x$,

(i) the new service charge for a call,

(ii) the new number of calls per day expected by the company,

(iii) the expected daily income of the company obtained from the service charge.

(b) In order to have a maximum daily income, how much should the company charge for each call?

(c) What is the maximum daily income in (*b*)?

(a) (i) the new service charge for a call = \$$(3 + x)$ per call

(ii) The new number of calls per day expected by the company

$= 4800 - 10x \times 120$

$= 4800 - 1200x$

(iii) The expected daily income of the company obtained from the service charge

$= \$(4800 - 1200x)(3 + x)$

(b) $\$(4800 - 1200x)(3 + x) = \$(1200)(12 + x - x^2)$

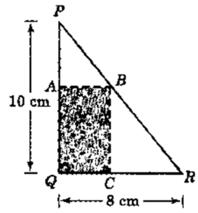
$= \$1200[12.25 - (0.25 - x + x^2)]$

$= \$14700 - 1200(x - 0.5)^2$

In order to have a maximum daily income, the company should charge \$3.5 for each call. ($x = 0.5$)

(c) the maximum daily income = \$14700

8. The figure shows a sheet of cardboard $PQR$ in the form of a right-angled triangle where $PQ$ = 10 cm and $QR$ = 8 cm. A rectangle $ABCQ$ is then cut away from the cardboard as shown.




(a) If $AB = x$ cm, express the area of $ABCQ$ in terms of $x$.

(b) Hence determine the maximum area of $ABCQ$ and its corresponding dimensions.

(a) $AB = x$ cm, $QC = x$ cm

$CR = (8 - x)$ cm

$\because \Delta BCR \sim \Delta PQR$

$$\frac{BC}{10} = \frac{(8-x)\text{cm}}{8}$$

$$BC = \frac{5}{4}(8-x)\text{ cm}$$

$$\text{Area} = \frac{5}{4}(8-x) \times x\text{ cm}^2$$

$$= \left(-\frac{5}{4}x^2 + 10x\right)\text{cm}^2$$

(b) $-0.8$ Area $= (x^2 - 8x)$ cm$^2$

$= [(x - 4)^2 - 16]$ cm$^2$

$$\text{Area} = \left[-\frac{5}{4}(x-4)^2 + 20\right]\text{cm}^2$$

When $x = 4$, the maximum area = 20 cm$^2$

$CR = (8 - 4)$ cm = 4 cm, $BC = \frac{5}{4}(8-4)$ cm = 5 cm

Dimension = 4 cm$\times$5 cm