Supplementary Exercise on Quadratic Equations

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Solve for the following equations.

1. $(x-4)(2x+8)(x+4)=0$
2. $(3x-5)(x^2-3x)=(3x-5)(2-3x)$
3. $2(21x^2-10)=11x$
4. $x(56x-15)=56$
5. $3x(-3x-2)=1$
6. $(x-4)(y+3)=0$
7. $x^2+1=0$
8. $(2x-1)^2+3=0$
9. $(3x-4)^2+(4x+6)^2=0$
10. $(3x-4)^2+(4y+6)^2=0$
11. $x^2+6(x-1)^2+3(x+1)^4=0$
12. $(2x+3y-1)^2+(4x-y-2)^2=0$
13. $x^2+(2x-3)^2=9$
14. $(x+1)^3+x^3=0$
15. $(3x+5)^4-(2x-7)^4=0$
16. $x+3=\frac{5}{2}+\frac{14}{x}$
17. $\frac{1}{(x-1)x}-\frac{1}{x(x+1)}=0$
18. $\frac{4}{x+5}-\frac{3}{x+4}=\frac{2}{x+3}-\frac{1}{x+2}$
19. $\frac{x^2+4x-1}{x^2+2x-3}=\frac{2}{3}+\frac{1}{x+3}$
20. $\frac{3x+5}{x+2}=\frac{4x+7}{x+2}-1$
21. $x^2-(a+\frac{1}{a})x+1=0$
22. $(x-5)(2x+3)=(a-5)(2a+3)$
23. $2^{2x}=5(2^x)-4$
24. $x^{\log x}=\frac{100}{x}$
25. $4\times5^{2x}+5^x-5=0$
26. $9^x+2\times3^x-8=0$
27. $20^x-401+400\times20^{-x}=0$
28. $3^x-24-3^{4-x}=0$
29. $(\log x)^2-3\log x+2=0$
30. $3(\log x)^2-10\log x=8$
31. $\log x+\log(2x+3)=\log 2$
32. $\log 2^x+\log(2^x-3)-\log 4=0$
33. By putting $y=x^2+3x$, solve $(x^2+3x)^2=2(x^2+3x+4)$.
34. $2(2y^2+8y)^2+7(2y^2+8y)-72=0$
35. (a) Solve $\begin{cases}5s+2t=8\\3s-5t=11\end{cases}$.

    (b) Hence solve $\begin{cases}5(x^2+x)+2(2y^2+3y)=8\\3(x^2+x)-5(2y^2+3y)=11\end{cases}$.
36. (a) Let $x=t+\frac{1}{t}$, express $t^2+\frac{1}{t^2}$ in terms of x.

    (b) Solve for t: $2(t^2+\frac{1}{t^2})-3(t+\frac{1}{t})-1=0$.
37. $3\tan^2\alpha+2\tan\alpha-1=0$
38. $8\sin^2 x-14\cos x-13=0$
39. $2\sin^2\theta-3\sin\theta\cos\theta+\cos^2\theta=0$
40. $2\sin^2 2x+3\sin 2x=2$
41. $\tan\theta=\frac{1+5\cos\theta}{\sin\theta}$
42. $2\sin^2 x\cdot\cos x=\cos x$
43. Let $t=\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\cdots}}}}$

    (a) Show that $t^2-t-2=0$

    (b) Hence find the value of t.
44. Let $t=\sqrt{10-3\sqrt{10-3\sqrt{10-3\sqrt{10-\cdots}}}}$

    (a) Find a suitable quadratic equation in t.

    (b) Hence find the value of t.
45. Let $x=\cfrac{2}{-1+\cfrac{2}{-1+\cfrac{2}{-1+\cdots}}}$

    (a) Show that $x^2-x-2=0$.

    (b) Hence find the value(s) of x.

1	4 or –4	2	$\frac{5}{3}, \sqrt{2}$ or $-\sqrt{2}$	3	$\frac{5}{6}$ or $-\frac{4}{7}$	4	$\frac{8}{7}$ or $-\frac{7}{8}$	5	$-\frac{1}{3}$ double root
6	$x = 4$ or $y = -3$	7	no solution	8	no solution	9	no solution	10	$x = \frac{4}{3}$ and $y = -\frac{3}{2}$
11	no solution	12	$x = \frac{1}{2}$ and $y = 0$	13	0 or $\frac{12}{5}$	14	$-\frac{1}{2}$	15	$\frac{2}{5}$ or –12
16	$x = \frac{7}{2}$ or –4	17	no solution	18	–1 or $-\frac{7}{2}$	19	–2	20	x can be any real number except –2
21	a or $\frac{1}{a}$	22	a or $-a + \frac{7}{2}$	23	0 or 2	24	$\frac{1}{100}$ or 10	25	0
26	$\frac{\log 2}{\log 3} = 0.631$	27	0 or 2	28	3	29	10 or 100	30	10000 or $10^{-2/3}$ (= 0.215)
31	$\frac{1}{2}$	32	2	33	–4, –2, –1, 1	34	–2, –2, $\frac{1}{2}$ $-\frac{9}{2}$	35	$t = -1$, $s = 2$ $(-2,-\frac{1}{2})$,(–2,–1), $(1,-\frac{1}{2})$,(1,–1)
36	(a) $x^2 - 2$ (b) $\frac{1}{2}$ or 2	37	18.43° or –45°	38	120°	39	26.57° or 45°	40	15°
41	120° or 70.53°	42	–45°, 45° or 90°	43	2	44	(a) $t^2+3t-10 = 0$ (b) $t = 2$	45	see below

1. $(x - 4)(2x + 8)(x + 4) = 0$
   $x = 4$ or $x = -4$ or $x = -4$
   $x = 4$ or –4
2. $(3x - 5)(x^2 - 3x) = (3x - 5)(2 - 3x)$
   $(3x - 5)(x^2 - 3x) - (3x - 5)(2 - 3x) = 0$
   $(3x - 5)[\, x^2 - 3x - (2 - 3x)] = 0$
   $x = \frac{5}{3}$ or $x^2 = 2$
   $x = \frac{5}{3}$ or $x = \sqrt{2}$ or $-\sqrt{2}$
3. $2\,(21x^2 - 10) = 11x$
   $42x^2 - 20 = 11x$
   $42x^2 - 11x - 20 = 0$
   $(6x - 5)(7x + 4) = 0$
   $x = \frac{5}{6}$ or $-\frac{4}{7}$
4. $x\,(56x - 15) = 56$
   $56x^2 - 15x - 56 = 0$
   $(7x - 8)(8x + 7) = 0$
   $x = \frac{8}{7}$ or $-\frac{7}{8}$
5. $3x\,(-3x - 2) = 1$
   $-9x^2 - 6x = 1$
   $9x^2 + 6x + 1 = 0$
   $x = -\frac{1}{3}$ double root
6. $(x - 4)\,(y + 3) = 0$
   $x = 4$ or $y = -3$
7. $x^2 + 1 = 0$
   L.H.S. $\geq 0 + 1 = 1$, R.H.S. $= 0$
   L.H.S. $\neq$ R.H.S.
   No solution
8. $(2x - 1)^2 + 3 = 0$
   L.H.S. $\geq 0 + 3 = 3$, R.H.S. $= 0$
   L.H.S. $\neq$ R.H.S.
   No solution
9. $(3x - 4)^2 + (4x + 6)^2 = 0$
   Sum of squares $= 0 \Rightarrow$ Each term $= 0$
   $\Rightarrow x = \frac{4}{3}$ and $x = -\frac{3}{2}$
   Which is a contradiction
   $\therefore$ No solution
10. $(3x - 4)^2 + (4y + 6)^2 = 0$
    Sum of squares $= 0 \Rightarrow$ Each term $= 0$
    $\Rightarrow x = \frac{4}{3}$ and $y = -\frac{3}{2}$
11. $x^2 + 6(x - 1)^2 + 3(x + 1)^4 = 0$
    Sum of squares $= 0 \Rightarrow$ Each term $= 0$
    $\Rightarrow x = 0$ and $x = 1$ and $x = -1$
    Which is a contradiction
    $\therefore$ No solution
12. $(2x + 3y - 1)^2 + (4x - y - 2)^2 = 0$
    Sum of squares $= 0 \Rightarrow$ Each term $= 0$
    $2x+3y-1 = 0 \cdots (1)$ and $4x-y-2 = 0 \cdots (2)$
    (1) +3(2): $14x - 7 = 0 \Rightarrow x = \frac{1}{2} \cdots (3)$
    Sub. (3) into (2): $4\cdot(\frac{1}{2}) - y - 2 = 0 \Rightarrow y = 0$
    $\therefore x = \frac{1}{2}$ and $y = 0$

13. $x^2 + (2x - 3)^2 = 9$
$x^2 + 4x^2 - 12x + 9 = 9$
$5x^2 - 12x = 0 \Rightarrow x = 0$ or $\frac{12}{5}$

14. $(x + 1)^3 + x^3 = 0$
$(x + 1 + x)[(x + 1)^2 - x(x + 1) + x^2] = 0$
$(2x + 1)(x^2 + 2x + 1 - x^2 - x + x^2) = 0$
$2x + 1 = 0$ or $x^2 + x + 1 = 0$
$x = -\frac{1}{2}$ or $\Delta = 1^2 - 4(1)(1) < 0$
$x = -\frac{1}{2}$ or no real solution
$x = -\frac{1}{2}$ only

15. $(3x + 5)^4 - (2x - 7)^4 = 0$
$[(3x+5)^2 - (2x-7)^2][(3x+5)^2 + (2x-7)^2] = 0$
$(x +12)(5x-2) = 0$ or $(x = -\frac{5}{3}$ and $x = \frac{7}{2})$
$(x = -12$ or $x = \frac{2}{5})$ or no solution
$x = -12$ or $x = \frac{2}{5}$

16. $x + 3 = \frac{5}{2} + \frac{14}{x}$
$2x^2 + 6x = 5x + 28$
$2x^2 + x - 28 = 0$
$(2x - 7)(x + 4) = 0$
$x = \frac{7}{2}$ or -4

17. $\frac{1}{(x-1)x} - \frac{1}{x(x+1)} = 0$
$\frac{(x+1)-(x-1)}{x(x-1)(x+1)} = 0$
$\frac{2}{x(x-1)(x+1)} = 0$
$2 = 0$
No solution

18. $\frac{4}{x+5} - \frac{3}{x+4} = \frac{2}{x+3} - \frac{1}{x+2}$
$\frac{4(x+4)-3(x+5)}{(x+4)(x+5)} = \frac{2(x+2)-(x+3)}{(x+2)(x+3)}$
$\frac{(x+1)}{(x+4)(x+5)} = \frac{(x+1)}{(x+2)(x+3)}$
$(x + 1)(x + 2)(x + 3) = (x + 1)(x + 4)(x +5)$
$(x + 1)(x+2)(x+3) - (x+1)(x + 4)(x +5) = 0$
$(x + 1)[(x+2)(x+3) - (x + 4)(x +5)] = 0$
$(x + 1)[x^2 + 5x + 6 - (x^2 + 9x + 20)] = 0$
$(x + 1)(-4x - 14) = 0$
$x = -1$ or $-\frac{7}{2}$

19. $\frac{x^2+4x-1}{x^2+2x-3} = \frac{2}{3} + \frac{1}{x+3}$, $x \neq 1$ and $x \neq -3$
$\frac{x^2+4x-1}{(x+3)(x-1)} = \frac{2}{3} + \frac{1}{x+3}$
$3(x^2 + 4x - 1) = 2(x^2 + 2x - 3) + 3(x - 1)$
$3x^2 + 12x - 3 = 2x^2 + 4x - 6 + 3x - 3$
$x^2 + 5x + 6 = 0$
$(x + 2)(x + 3) = 0$
$x = -2$ or $x = -3$ (rejected)
$x = -2$ only

20. $\frac{3x+5}{x+2} = \frac{4x+7}{x+2} - 1$, $x \neq -2$
$3x + 5 = 4x + 7 - (x + 2)$
$3x + 5 = 3x + 5$
which is always true
x can be any real number except -2.

21. $x^2 - (a + \frac{1}{a})x + 1 = 0$
$(x - a)(x - \frac{1}{a}) = 0$
$x = a$ or $x = \frac{1}{a}$

22. $(x - 5)(2x + 3) = (a - 5)(2a + 3)$
$2x^2 - 7x - 15 = 2a^2 - 7a - 15$
$2x^2 - 2a^2 - 7x + 7a = 0$
$2(x + a)(x - a) - 7(x - a) = 0$
$(x - a)[2(x + a) - 7] = 0$
$x = a$ or $-a + \frac{7}{2}$

23. $2^{2x} = 5(2^x) - 4$
$(2^x)^2 - 5(2^x) + 4 = 0$
$(2^x - 1)(2^x - 4) = 0$
$2^x = 1$ or 2^2
$x = 0$ or 2

24. $x^{\log x} = \frac{100}{x}$
$\log(x^{\log x}) = \log(\frac{100}{x})$
$\log x (\log x) = \log 100 - \log x$
$(\log x)^2 + \log x - 2 = 0$
$(\log x + 2)(\log x - 1) = 0$
$\log x = -2$ or 1
$x = 10^{-2}$ or 10^1
$x = \frac{1}{100}$ or 10

25. $4\times5^{2x} + 5^x - 5 = 0$
$4\times(5^x)^2 + 5^x - 5 = 0$
$(4\times5^x + 5)(5^x - 1) = 0$
$5^x = -\frac{5}{4}$ (rejected) or 1
$x = 0$

26. $9^x + 2\times3^x - 8 = 0$
$(3^x)^2 + 2\times3^x - 8 = 0$
$(3^x - 2)(3^x + 4) = 0$
$3^x = 2$ or -4 (rejected)
$\log 3^x = \log 2$
$x \log 3 = \log 2 \Rightarrow x = \dfrac{\log 2}{\log 3} = 0.631$

27. $20^x - 401 + 400\times20^{-x} = 0$
$20^x - 401 + 400\times\dfrac{1}{20^x} = 0$
$(20^x)^2 - 401\times20^x + 400 = 0$
$(20^x - 1)(20^x - 400) = 0$
$20^x = 1$ or 20^2
$x = 0$ or 2

28. $3^x - 24 - 3^{4-x} = 0$
$3^x - 24 - \dfrac{3^4}{3^x} = 0$
$(3^x)^2 - 24\times3^x - 81 = 0$
$(3^x - 27)(3^x + 3) = 0$
$3^x = 3^3$ or -3 (rejected)
$x = 3$

29. $(\log x)^2 - 3\log x + 2 = 0$
$(\log x - 1)(\log x - 2) = 0$
$\log x = 1$ or 2
$x = 10^1$ or 10^2
$x = 10$ or 100

30. $3(\log x)^2 - 10\log x = 8$
$3(\log x)^2 - 10\log x - 8 = 0$
$(\log x - 4)(3\log x + 2) = 0$
$\log x = 4$ or $-\dfrac{2}{3}$
$x = 10^4$ or $10^{-\frac{2}{3}}$
$x = 10000$ or 0.215 (correct to 3 sig. fig.)

31. $\log x + \log(2x + 3) = \log 2$
$\log x(2x + 3) = \log 2$
$2x^2 + 3x = 2$
$2x^2 + 3x - 2 = 0$
$(2x - 1)(x + 2) = 0$
$x = \dfrac{1}{2}$ or -2

Put $x = \dfrac{1}{2}$ into the original equation

$\text{LHS} = \log\dfrac{1}{2} + \log(1 + 3)$

$= \log\left(\dfrac{1}{2}\times4\right) = \log 2 = \text{RHS}$

Put $x = -2$ into the original equation
$\text{LHS} = \log(-2) + \log(-1 + 3)$
which is undefined, rejected
$\therefore x = \dfrac{1}{2}$

32. $\log 2^x + \log(2^x - 3) - \log 4 = 0$
$\log 2^x(2^x - 3) = \log 4$
$(2^x)^2 - 3(2^x) - 4 = 0$
$(2^x - 4)(2^x + 1) = 0$
$2^x = 4$ or -1 (rejected)
$x = 2$

33. $(x^2 + 3x)^2 = 2(x^2 + 3x + 4)$
Let $y = x^2 + 3x$, the equation becomes
$y^2 = 2(y + 4)$
$y^2 - 2y - 8 = 0$
$(y - 4)(y + 2) = 0$
$y = 4$ or -2
When $y = 4$, $x^2 + 3x = 4$
$x^2 + 3x - 4 = 0$
$(x - 1)(x + 4) = 0$
$x = 1$ or -4
When $y = -2$, $x^2 + 3x = -2$
$x^2 + 3x + 2 = 0$
$(x + 1)(x + 2) = 0$
$x = -1$ or -2
$\therefore x = -4, -2, -1, 1$

34. $2(2y^2 + 8y)^2 + 7(2y^2 + 8y) - 72 = 0$
Let $u = 2y^2 + 8y$, the equation becomes
$2u^2 + 7u - 72 = 0$
$(2u - 9)(u + 8) = 0$
$u = \dfrac{9}{2}$ or -8

When $u = \dfrac{9}{2}$, $2y^2 + 8y = \dfrac{9}{2}$
$4y^2 + 16y - 9 = 0$
$(2y + 9)(2y - 1) = 0$
$y = -\dfrac{9}{2}$ or $\dfrac{1}{2}$
When $y = -8$, $2y^2 + 8y = -8$
$y^2 + 4y + 4 = 0$
$(y + 2)^2 = 0$
$\therefore y = -\dfrac{9}{2}, -2$ or $\dfrac{1}{2}$

35. (a) Solve $\begin{cases} 5s + 2t = 8 & \cdots(1) \\ 3s - 5t = 11 & \cdots(2) \end{cases}$
3(1): $15s + 6t = 24$
5(2): $15s - 25t = 55$
3(1) – 5(2): $31t = -31$
$t = -1$
Sub. $t = -1$ into (2): $3s + 5 = 11$
$s = 2$
$\therefore t = -1$ or $s = 2$

(b) $\begin{cases} 5(x^2 + x) + 2(2y^2 + 3y) = 8 \\ 3(x^2 + x) - 5(2y^2 + 3y) = 11 \end{cases}$
Let $s = x^2 + x$, and $t = 2y^2 + 3y$.
$\begin{cases} 5s + 2t = 8 & \cdots(1) \\ 3s - 5t = 11 & \cdots(2) \end{cases}$
By (a), $s = 2$, $t = -1$

$x^2 + x = 2,\ 2y^2 + 3y = -1$
$x^2 + x - 2 = 0,\ 2y^2 + 3y + 1 = 0$
$(x + 2)(x - 1) = 0,\ (2y + 1)(y + 1) = 0$
$x = -2$ or 1; $y = -\frac{1}{2}$ or -1

$(x,y)=(-2,-\frac{1}{2}),(-2,-1),(1,-\frac{1}{2}),(1,-1).$

36. (a) $x = t + \frac{1}{t} \Rightarrow x^2 = t^2 + 2 + \frac{1}{t^2}$

$t^2 + \frac{1}{t^2} = x^2 - 2$

(b) $2(t^2 + \frac{1}{t^2}) - 3(t + \frac{1}{t}) - 1 = 0$

Let $x = t + \frac{1}{t}$, by (a), $t^2 + \frac{1}{t^2} = x^2 - 2$
It becomes $2(x^2 - 2) - 3x - 1 = 0$
$2x^2 - 3x - 5 = 0$
$(2x - 5)(x + 1) = 0$
$x = \frac{5}{2}$ or -1

When $x = \frac{5}{2}$, $t + \frac{1}{t} = \frac{5}{2}$
$2t^2 + 2 = 5t$
$2t^2 - 5t + 2 = 0$
$(2t - 1)(t - 2) = 0$
$t = \frac{1}{2}$ or 2

When $x = -1$, $t + \frac{1}{t} = -1$
$t^2 + 1 = -t$
$t^2 + t + 1 = 0$
$\Delta = b^2 - 4ac = 1^2 - 4 = -3 < 0$
$\therefore t = \frac{1}{2}$ or 2

37. $3\tan^2\alpha + 2\tan\alpha - 1 = 0$
$(3\tan\alpha - 1)(\tan\alpha + 1) = 0$
$\tan\alpha = \frac{1}{3}$ or -1
$\alpha = 18.43°$ or $-45°$

38. $8\sin^2 x - 14\cos x - 13 = 0$
$8(1 - \cos^2 x) - 14\cos x - 13 = 0$
$8\cos^2 x + 14\cos x + 5 = 0$
$(4\cos x + 5)(2\cos x + 1) = 0$
$\cos x = -\frac{5}{4}$ (rejected) or $-\frac{1}{2}$
$x =$ $120°$

39. $2\sin^2\theta - 3\sin\theta\cos\theta + \cos^2\theta = 0$
$(2\sin\theta - \cos\theta)(\sin\theta - \cos\theta) = 0$
$2\sin\theta = \cos\theta$ or $\sin\theta = \cos\theta$
$\tan\theta = \frac{1}{2}$ or 1
$\theta = 26.57°$ or $45°$

40. $2\sin^2 2x + 3\sin 2x = 2$
$2\sin^2 2x + 3\sin 2x - 2 = 0$
$(2\sin 2x - 1)(\sin 2x + 2) = 0$
$\sin 2x = \frac{1}{2}$ or -2 (rejected)
$2x = 30° \Rightarrow x = 15°$

41. $\tan\theta = \frac{1 + 5\cos\theta}{\sin\theta}$
$\tan\theta\sin\theta = 1 + 5\cos\theta$
$\frac{\sin^2\theta}{\cos\theta} = 1 + 5\cos\theta$
$\sin^2\theta = \cos\theta + 5\cos^2\theta$
$1 - \cos^2\theta = \cos\theta + 5\cos^2\theta$
$6\cos^2\theta + \cos\theta - 1 = 0$
$(2\cos\theta + 1)(3\cos\theta - 1) = 0$
$\cos\theta = -\frac{1}{2}$ or $\frac{1}{3}$
$\theta = 120°$ or $70.53°$

42. $2\sin^2 x \cdot \cos x = \cos x$
$2\sin^2 x \cdot \cos x - \cos x = 0$
$\cos x\,(2\sin^2 x - 1) = 0$
$\cos x = 0$ or $\sin x = \frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}}$
$x = 90°$, $45°$ or $-45°$

43. (a) $t^2 = 2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{\cdots}}}}$
$t^2 = 2 + t$
$t^2 - t - 2 = 0$
(b) $t = 2$ (reject $t = -1$)

44. (a) Let $t = \sqrt{10 - 3\sqrt{10 - 3\sqrt{10 - 3\sqrt{10 - \cdots}}}}$
$t^2 = 10 - 3\sqrt{10 - 3\sqrt{10 - 3\sqrt{10 - \cdots}}}$
$t^2 = 10 - 3t$
$t^2 + 3t - 10 = 0$
(b) $(t - 2)(t + 5) = 0$
$t = 2$ or -5 (rejected)

45. (a) $\frac{2}{x} = -1 + \frac{2}{-1 + \frac{2}{-1 + \cdots}}$

$\frac{2}{x} = -1 + x \Rightarrow 2 = -x + x^2$
$x^2 - x - 2 = 0$
(b) $(x - 2)(x + 1) = 0$
$\Rightarrow x = 2$ or -1
Consider the following sequences
(i) $2, \frac{2}{-1 + 2}, \frac{2}{-1 + \frac{2}{-1 + 2}}, \cdots$
Each term is equal to 2.

(ii) $\frac{2}{-1}, \frac{2}{-1+\frac{2}{-1}}, \frac{2}{-1+\frac{2}{-1+\frac{2}{-1}}}, \cdots$

This sequence is equivalent to $-2, -\frac{2}{3}, -\frac{6}{5}, \cdots$

It converges to –1. However, x cannot converges to two different numbers. $\therefore$ There is no solution.