Triangle Inequality

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HKCEE Additional Mathematics 1987 Paper 1 Q3

Let z be a complex number. Then $|z|^2 = z\,\bar{z}$

$\text{Re}(z) = \frac{1}{2}(z + \bar{z})$; $\text{Im}(z) = \frac{1}{2i}(z - \bar{z})$

If $z = a + bi$, a, b are real numbers,

$|z| = \sqrt{a^2 + b^2} \ge a = \text{Re}(z)$

If z_1, z_2 are any complex numbers,

Let $z = z_1\, z_2$

$|z| \ge \text{Re}(z) \Rightarrow |z_1\, z_2| \ge \frac{1}{2}(z + \bar{z}) = \frac{1}{2}(z_1 z_2 + \bar{z}_1 \bar{z}_2)$

$z_1 z_2 + \bar{z}_1 \bar{z}_2 \le 2|z_1||z_2|$

Replace z_2 by $\bar{z}_2$ $\Rightarrow$ $z_1 \bar{z}_2 + \bar{z}_1 z_2 \le 2|z_1||z_2|$

$|z_1 + z_2|^2 = (z_1 + z_2)(\bar{z}_1 + \bar{z}_2) = z_1\,\bar{z}_1 + \bar{z}_1\, z_2 + z_1\,\bar{z}_2 + z_2\,\bar{z}_2$

$\le |z_1|^2 + 2|z_1||z_2| + |z_1|^2$

$= (|z_1| + |z_2|)^2$

$|z_1 + z_2| \le |z_1| + |z_2|$

Inductively $|z_1 + z_2 + \ldots + z_n| \le |z_1| + |z_2| + \ldots + |z_n|$.