Angle bisector, circumcentre and altitude

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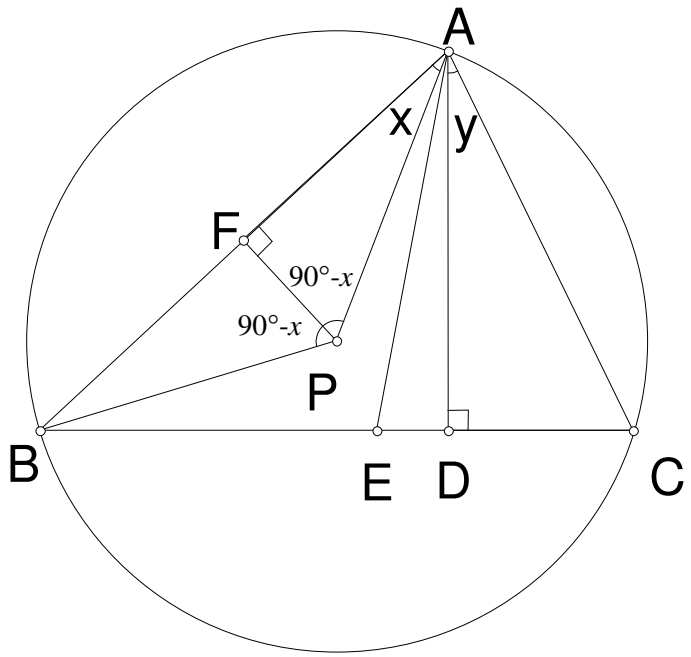
ABC is an acute-angled triangle with circumcentre P. If AD is the altitude to BC, **prove that the bisector of angle A bisects $\angle DAP$.**



AE is the angle bisector.

Then $\angle BAE = \angle CAE$ ····· (1)

Let $\angle BAP = x$, $\angle CAD = y$.

Since P is the circumcentre, if F is the mid point of AB, then PF is the perpendicular bisector of AB.

Draw the circumcircle ABC.

$\angle APF = 90° - x$ $\angle$ sum of Δ

$\angle BPF = 90° - x$ $\Delta APF \cong \Delta BPF$

$\therefore \angle APB = 180° - 2x$

$\angle ACB = 90° - x$ $\angle$ at circumference half $\angle$ at centre

$90° - x + y = 90°$ $\angle$ sum of ΔADC

$x = y$ ····· (2)

Combine (1) and (2),

$\angle DAE = \angle PAE$

Therefore, the bisector of angle A bisects $\angle DAP$.