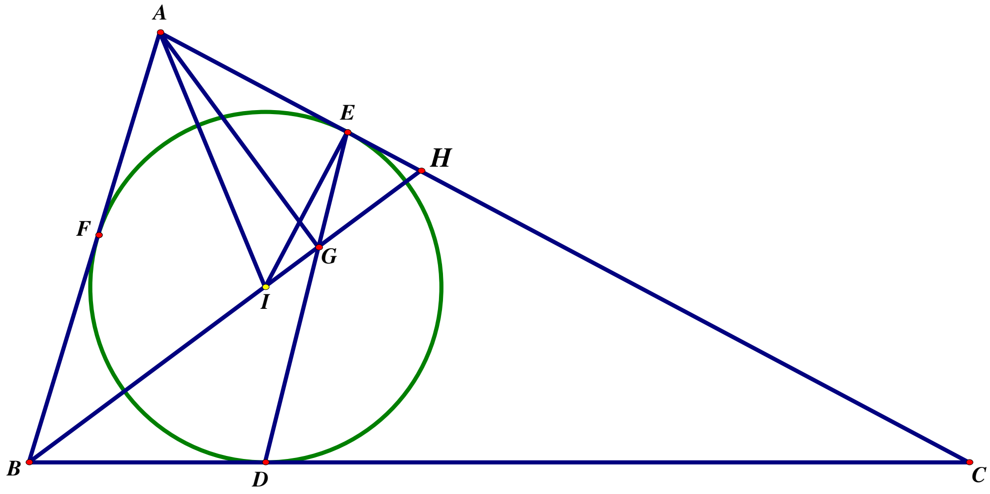
Incentre Example 2

Created by Francis Hung Last updated: 03 September 2021



In the figure, the incircle with centre I touches the triangle ABC at D, E and F. BI is produced to meet AC at H. DE intersects BH at G. To prove $AG \perp BH$.

Let $\angle A = 2x$, $\angle B = 2y$, $\angle C = 2z$.

$2x + 2y + 2z = 180°$ ($\angle$s sum of Δ) $\Rightarrow x + y + z = 90°$ $\cdots\cdots$ (1)

$\angle IAH = x$, $\angle IBD = y$ (by the definition of incentre, IA bisects $\angle A$, IB bisects $\angle B$)

$CD = CE$ (tangent from ext. point)

$\angle DCE = \angle ECD = (180° - 2z) \div 2 = 90° - z$ (base $\angle$s. isos. Δ, $\angle$ sum of Δ)

In ΔBDG, $\angle BGD = \angle CDG - \angle DBG = 90° - z - y = x$ (ext. $\angle$ of Δ, and by (1))

$\angle EGH = \angle BGD = x$ (vert. opp. $\angle$s)

$\therefore \angle IAH = x = \angle EGH$ $\cdots\cdots$ (2)

$\therefore A, E, G, I$ are concyclic (ext. $\angle$ = int. opp. $\angle$)

$\angle AEI = 90°$ (tangent $\perp$ radius)

$\angle AGI = \angle AEI = 90°$ ($\angle$s in the same segment)

$\therefore AG \perp BH$. The proof is completed.

Exercise: If DE intersects BI at G outside ΔABC, prove that $AG \perp BH$.