Inscribed circle

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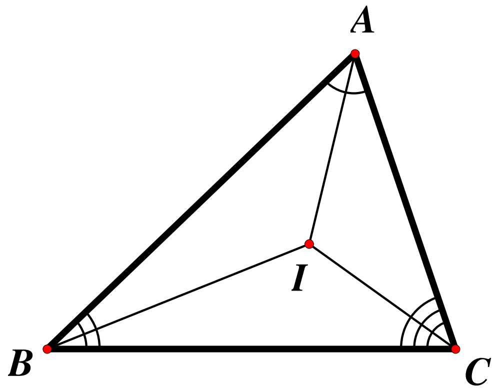
The 3 **angle bisectors** of a triangle are **concurrent** at a point called the **inscribed centre**.

Let AI, BI, CI be the angle bisectors.

The **theorem** says that $\boldsymbol{AI}$, $\boldsymbol{BI}$ and $\boldsymbol{CI}$ meet at $\boldsymbol{I}$.

To prove this **theorem**, suppose the **angle bisectors** BI and CI intersect at I. Join AI. Try to show that AI is **an angle bisector**.

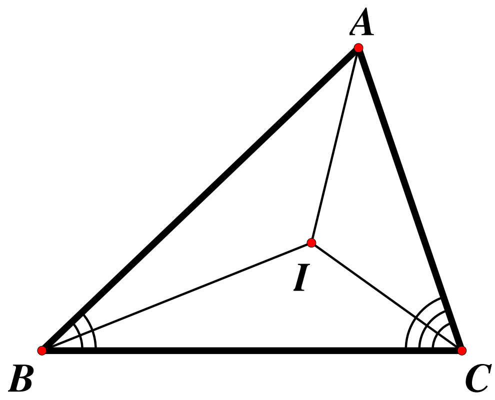


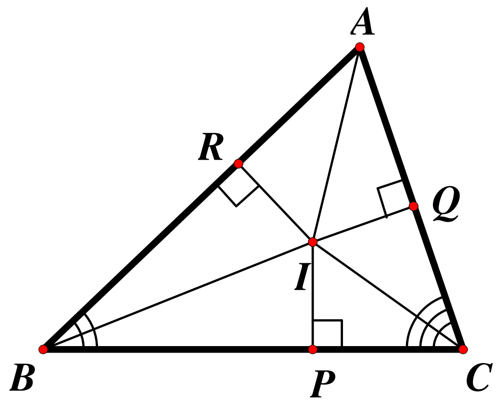
From I, draw $IP \perp BC$, $IQ \perp AC$, $IR \perp AB$

$\angle ICP = \angle ICQ$	(angle bisector)
$\angle IPC = \angle IQC = 90°$	(construction)
$IC = IC$	(common side)
$\therefore \Delta IPC \cong \Delta IQC$	(A.A.S.)
$\angle IBP = \angle IBR$	(angle bisector)
$\angle IPB = \angle IRB = 90°$	(construction)
$IB = IB$	(common side)
$\therefore \Delta IBP \cong \Delta IBR$	(A.A.S.)
$IQ = IP = IR$	(corr. sides of $\cong \Delta$s)
$\angle IQA = \angle IRA = 90°$	(construction)
$AI = AI$	(common side)
$\therefore \Delta IRA \cong \Delta IQA$	(R.H.S.)
$\therefore \angle IAR = \angle IAQ$	(corr. $\angle$s $\cong \Delta$s)

$\therefore$ AI is **an angle bisector.**

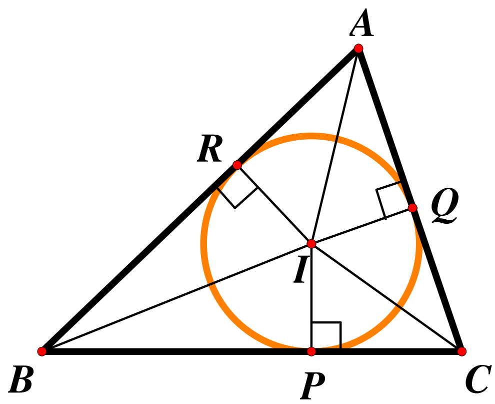
This proves the theorem.





According to **this theorem**, **we can draw a circle** with I as the centre, $IP = IQ = IR$ as the radius. The circle is called the **inscribed circle** or **incircle** in short. The centre is called the **inscribed centre** or **incentre** in short.

The radius is insribed radius or inradius in short.



We can find the **radius (*r*) of the inscribed circle** in terms of the **sides of triangle *ABC*.**

Area of ABC = Area of IBC + Area of IAC + Area of IAB

By **Heron's Formula**, $s=\frac{1}{2}(a+b+c)$, the area $=\sqrt{s(s-a)(s-b)(s-c)}$

$$\sqrt{s(s-a)(s-b)(s-c)} = \frac{1}{2}ar+\frac{1}{2}br+\frac{1}{2}cr$$

$$\sqrt{s(s-a)(s-b)(s-c)} = sr.$$

$$r=\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$