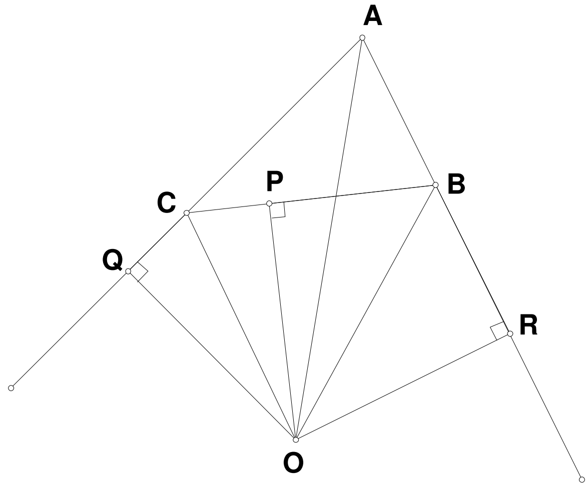
# Escribed circle

Created by Francis Hung Last updated: 03 September 2021

**Two exterior angle bisectors** and **one interior angle bisector** of a triangle are **concurrent** at a point. We can use this point as a centre and a suitable radius to draw an **escribed circle**.

Let $CO$, $BO$ be the **exterior angle bisectors** meet at $O$. Join $AO$. Try to prove that $AO$ is **an interior angle bisector.**

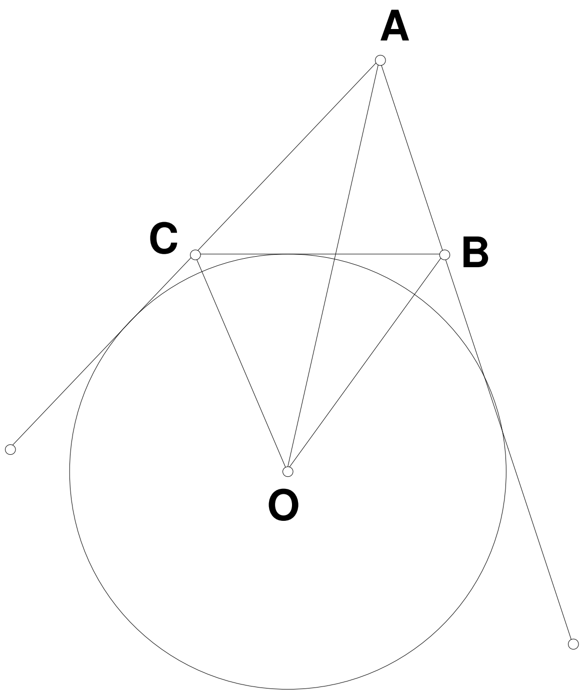
Let $P$, $Q$, $R$ be the **feet of perpendiculars** from $O$ onto the sides $BC$, $AC$ produced and $AB$ produced respectively.




| | |
|---|---|
| $\angle OQC = \angle OPC$ | (by construction) |
| $OC = OC$ | (common sides) |
| $\angle OCQ = \angle OCP$ | (angle bisector) |
| $\Delta OCQ \cong \Delta OCP$ | (A.A.S.) |
| $\angle ORB = \angle OPB$ | (construction) |
| $OB = OB$ | (common side) |
| $\angle OBR = \angle OBP$ | (angle bisector) |
| $\Delta OBR \cong \Delta OBP$ | (A.A.S.) |
| $\therefore\ OQ = OP = OR$ | (corr. sides, $\cong\Delta$s) |
| $OA = OA$ | (common side) |
| $\angle OQA = \angle ORA$ | (construction) |
| $\Delta OAQ \cong \Delta OAR$ | (R.H.S.) |
| $\therefore\ \angle OAQ = \angle OAR$ | (corr. $\angle$s, $\cong\Delta$s) |

$\therefore AO$ is **an interior angle bisector.**

Hence we can use $O$ as **centre** and $OP = OQ = OR$ as **radii** to draw a **circle** passes through $P$ , $Q$ , $R$ .




To find **the radius** $r$ of the **escribed circle** in terms of **the sides** of $\Delta ABC$ ,

Area of $\Delta ABC$ = area of $\Delta AOC$ + area of $\Delta AOB$ − area of $\Delta COB$

$$\sqrt{s(s-a)(s-b)(s-c)} = \frac{1}{2}br + \frac{1}{2}cr - \frac{1}{2}ar = \frac{r}{2}(b+c-a) = \frac{r}{2}(2s-2a) = r(s-a)$$

$$r = \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s-a} = \sqrt{\frac{s(s-b)(s-c)}{s-a}}$$