20°-80°-80° triangle

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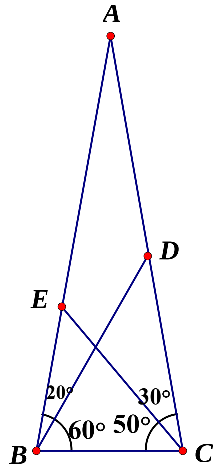
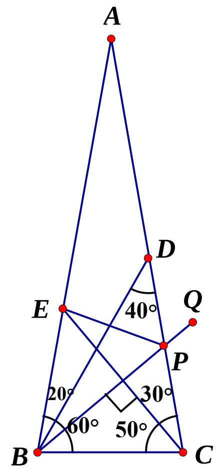
In ΔABC, E is a point on side AB and D is a point on side AC such that $\angle ABD = 20°$, $\angle DBC = 60°$, $\angle ACE = 30°$ and $\angle ECB = 50°$. Find $\angle EDB$.



Solution:



$\angle BEC = 50°$ (∠ sum of Δ)

$\angle BDC = 40°$ (∠ sum of Δ)

Let P be a point on AC s.t. $BP \perp CE$. Draw $BPQ \perp CE$.

$\because BC = BE$ (sides opp. eq. ∠s)

$\therefore$ BPQ is the $\perp$ bisector of CE.

$\Delta BPC \cong \Delta BPE$ (S.A.S.)

$\angle EBP = 40° \Rightarrow \angle DBP = 20°$

$\angle BPC = \angle BPE = 180° - \angle PBC - \angle PCB$ (∠ sum of Δ)

$= 180° - 40° - 80° = 60°$

$\angle DPQ = 60°$ (vert. opp. ∠s)

$\angle DPE = 60°$ (adj. ∠s on st. lines.)

$\therefore$ DP is the exterior angle bisector of $\angle QPE$

Also BD is the interior angle bisector of $\angle PBE$

$\therefore D$ is the centre of the escribed circle BEP.

$\therefore DE$ is the exterior angle bisector of $\angle AEP$.

$\angle AED = \frac{1}{2}\angle AEP = \frac{1}{2}\angle BCP$ (adj. ∠s on st. line.)

$= \frac{1}{2}\left(180^\circ - 50^\circ - 30^\circ\right) = 50°$

$\angle EDB = 50° - 20° = 30°$ (ext. ∠ on ΔBDE)