Problem on equilateral triangle

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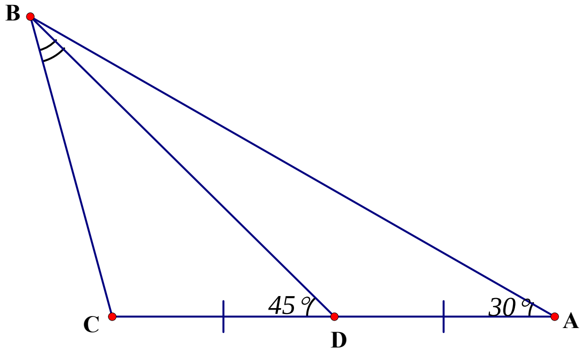
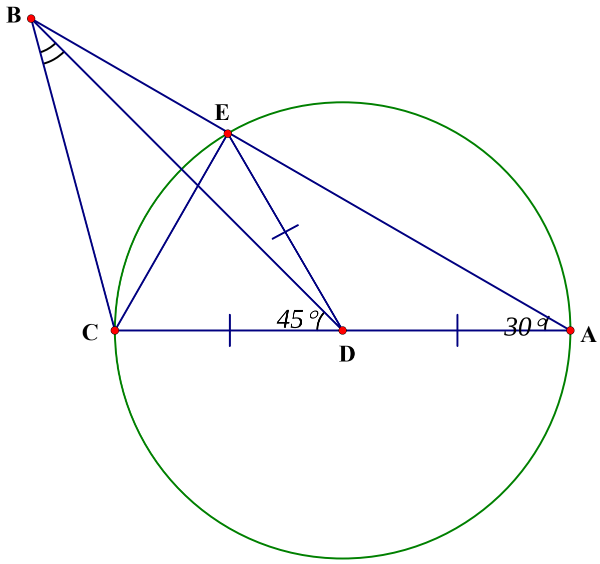


In ΔABC, D is the mid-point of AC. $\angle BAC = 30°$, $\angle BDC = 45°$. Find $\angle CBD$.

Draw a circle $\odot(D, DA)$, cutting AB at E. Join CE.



$DA = DE = DC$ (radii)

$\angle ABD = \angle BDC - \angle BAD$ (ext. $\angle$ of Δ)

$= 45° - 30° = 15°$

$\angle AED = \angle DAE = 30°$ (base $\angle$s isos. Δ)

In ΔADE,

$\angle CDE = 30° + 30° = 60°$ (ext. $\angle$ of Δ)

In ΔCDE,

$\angle DCE = \angle DEC$ (base $\angle$s isos. Δ)

$= \frac{180° - 60°}{2}$ ($\angle$ sum of Δ)

$= 60°$

$\therefore$ ΔCDE is an equilateral triangle

$CE = CD = DE$ (property of equialteral triangle)

$\angle BDE = 60° - 45° = 15° = \angle ABD$

$\therefore$ $BE = DE$ (sides opp. equal angles)

$\therefore$ $CE = DE = BE$

$\angle CEA = 60° + 30° = 90°$

$\therefore$ ΔCDE is an right-angled isosceles triangle

$\angle CBE = \angle BCE = 45°$ (base $\angle$s, isos. Δ, $\angle$ sum of Δ)

$\angle CBD = \angle CBE - \angle DBE$

$= 45° - 15°$

$= 30°$