Triangle Problem 1

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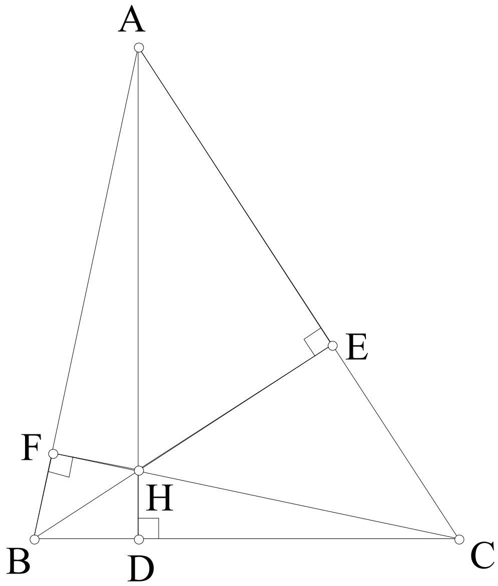
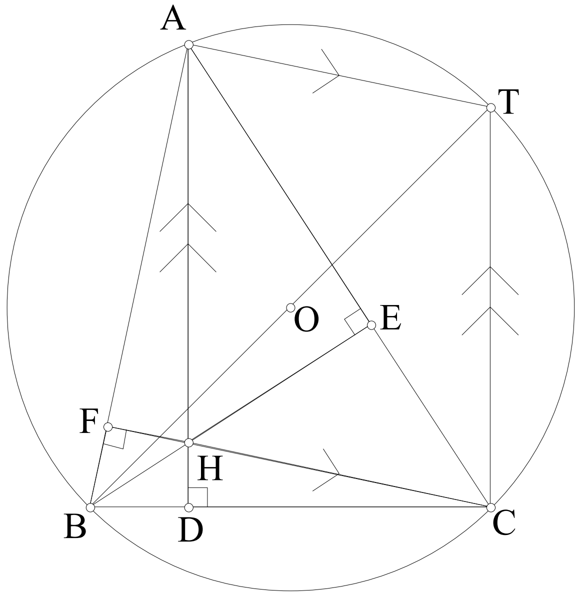
In the figure, H is the orthocentre of the triangle ABC. Given that $AH = BC$. Prove that $\angle BAC = 45°$





Let AD, BE, CF be the altitudes.
Let O be the circumcentre of the triangle. Let BT be the diameter.

$\angle BCT = 90°$	($\angle$ in semi-circle)
$\angle ADC = 90°$	(AD = altitude)
$\therefore AH \parallel CT$	
$\angle BAT = 90°$	($\angle$ in semi-circle)
$\angle BFC = 90°$	(CF = altitude)
$\therefore AT \parallel CH$	
$AHCT$ is a parallelogram.	
$AH = CT$	(opp. sides of //-gram)
$BC = AH$	(given)
$\therefore BC = CT$ and $\angle BCT = 90°$	
$\therefore \angle BTC = 45°$	($\angle$ sum of isos. Δ)
$\therefore \angle BAC = 45°$	($\angle$s in the same segment)

Method 2

$AH = BC$	(given)
$\angle AEH = 90° = \angle BEC$	($\because$ BE is an altitude)
Let $\angle BHD = \theta$, then $\angle AHE = \theta$	(vert. opp. $\angle$s)
In ΔAEH, $\angle EAH = 90° - \theta$	($\angle$ sum of Δ)
In ΔBDH, $\angle DBH = 90° - \theta = \angle EAH$	($\angle$ sum of Δ)
$\therefore \angle BHD = \angle CAD = \theta$	
$\angle CAD = \angle CBE = 90° - \theta$	
$\Delta BCE \cong \Delta AHE$	(A.A.S.)
$BE = AE$	(corr. sides, $\cong$ Δs)
ΔABE is a right-angled isosceles triangle	
$\angle ABE = \angle BAE$	(corr. $\angle$s, $\cong$ Δs)
$= \dfrac{180^\circ - 90^\circ}{2} = 45°$	($\angle$ sum of Δ)