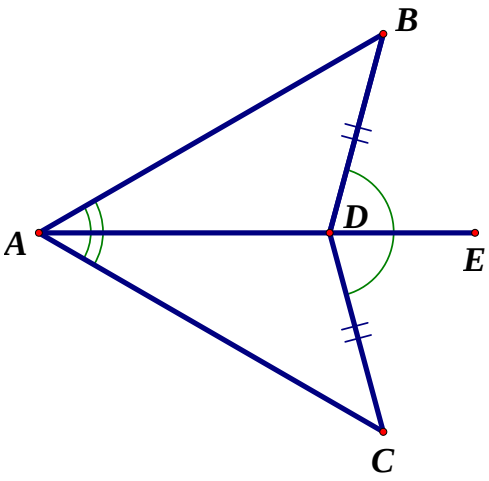
S.S.A.

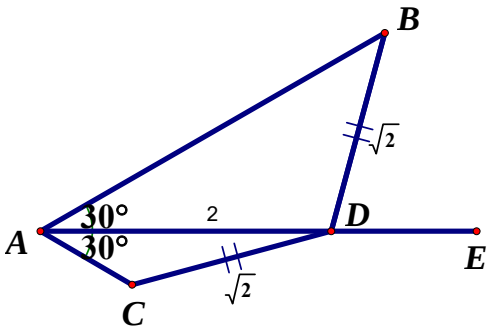
Created by Mr. Francis Hung on 20210902 Last updated: 2021-09-02

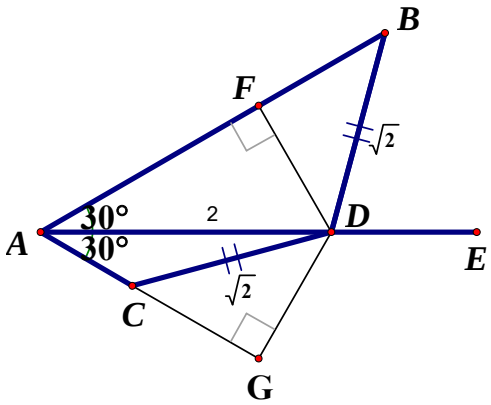
In the figure, $\angle BAD = \angle CAD$, $BD = CD$ and AD is produced to E.



Question: Will DE bisect $\angle BDC$?

Answer: The diagram misleads you. It is possible that the diagram is as follows:





In the figure, $\angle BAD = \angle CAD = 30°$, $AD = 2$, $BD = CD = \sqrt{2}$ and AD is produced to E.

Draw the altitudes $DF \perp AB$ and $DG \perp AC$ produced.

Then $DF = AD \sin 30° = 2 \sin 30° = 1$, $DG = AD \sin 30° = 2 \sin 30° = 1$

$AF = 2 \cos 30° = \sqrt{3} = AG$

In ΔBDG and ΔCDG, $BF^2 + 1^2 = 2$ and $CG^2 + 1^2 = 2$

$\Rightarrow BF = 1$ and $CG = 1$

$\Rightarrow \Delta BDF$ and ΔCDG are right angled isosceles triangles

$\Rightarrow \angle BDF = 45°$ and $\angle CDG = 45°$

$\Rightarrow \angle ADB = 60° + 45° = 105°$ and $\angle ADC = 60° - 45° = 15°$

$\therefore \angle BDE = 180° - 105° = 75°$ and $\angle CDE = 180° - 15° = 165°$

$\because \angle BDE \neq \angle CDE$

$\therefore AD$ may not bisect $\angle BDC$.