Pentagon problem

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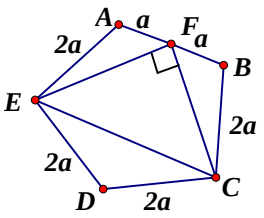
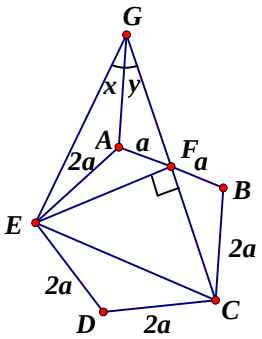
Last updated: 21 April 2011

Given a convex pentagon $ABCDE$ with each side = $2a$.

F is the mid point of AB. $\angle CFE = 90°$. To prove $\angle AED + \angle BCD = 180°$.



Proof: Produce CF to G so that $CF = FG$. Join EG.



$\Delta AFG \cong \Delta BFC$	(S.A.S.)
Join CE. Also, $\Delta EFG \cong \Delta EFC$	(S.A.S.)
So $AG = BC = 2a$, $EG = EC$	(corr. $\angle$s $\cong \Delta$s)
$\Delta AEG \cong \Delta DEC$	(S.S.S.)
Let $\angle AGE = x$, $\angle AGF = y$.	
Then $\angle AEG = x$	(base $\angle$s. isos. Δ)
$\angle DCE = x = \angle DEC$	(corr. $\angle$s $\cong \Delta$s)
$\angle ECF = x + y$	(corr. $\angle$s $\cong \Delta$s)
$\angle BCF = y$ (corr. $\angle$s $\cong \Delta$s)	

$\angle AED + \angle BCD = (x + \angle CEG - \angle AEG) + (x + \angle ECF + \angle BCF)$

$= (x + \angle CEG - x) + (x + x + y + y)$

$= \angle CEG + \angle ECF + \angle EGF$

$= 180°$ ($\angle$ sum of ΔCEG)