Folding Paper Problem 2

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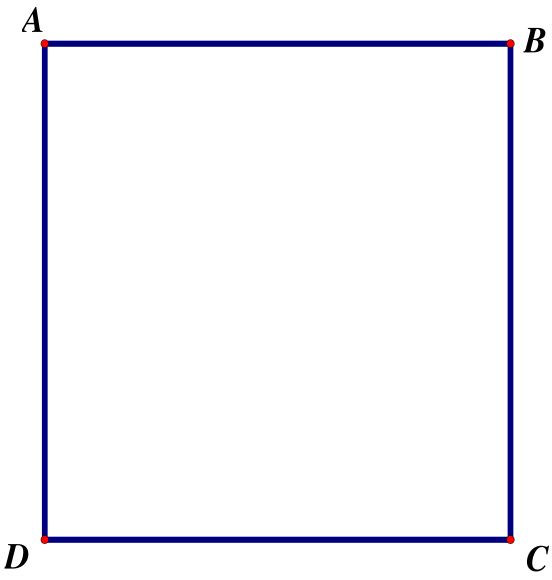

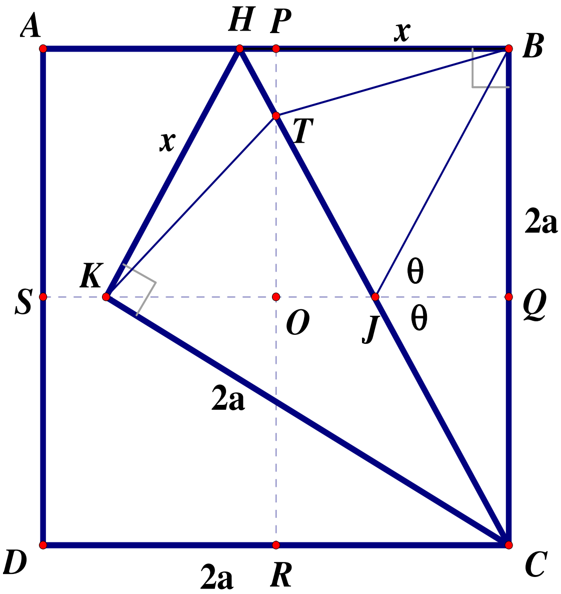

In the figure, $ABCD$ is a square with side = $2a$.

PR and QS are the lines of symmetry of the square intersecting at O .

The corner at B is folded to K on SQ. i.e. $\Delta BCH \cong \Delta KCH$

To prove that $\angle BCH = 30°$

Suppose that CH intersects PR at T and SQ at J .

Then it can be easily proved that $\Delta KHT \cong \Delta BHT$ (S.A.S.); $\Delta KCJ \cong \Delta BCJ$ (S.A.S.)

Hence $\Delta KJT \cong \Delta BJT$ (S.S.S.)

Also, it is easy to prove that $\Delta CJQ \cong \Delta BJQ$ (S.A.S.)

$\therefore$ $\angle CJQ = \angle BJQ = \theta$ (corr. $\angle$s, $\cong$ Δs)

$\angle CJK = \angle CJB = 2\theta$ (corr. $\angle$s, $\cong$ Δs)

$\angle KJT = \angle CJQ = \theta$ (vert. opp. $\angle$s)

$\angle BJT = \angle KJT = \theta$ (corr. $\angle$s, $\cong$ Δs)

At J, $\theta + \theta + 2\theta + \theta + \theta = 360°$ ($\angle$s at a point)

$\Rightarrow \theta = 60°$

$\angle BJC = 2\theta = 120°$

$\because$ ΔBJC is an isosceles triangle

$\therefore$ $\angle BCJ = \angle CBJ = 30°$ (base $\angle$s isosceles Δ)