# Fold paper problem 3 (To construct $\sqrt{ab}$)

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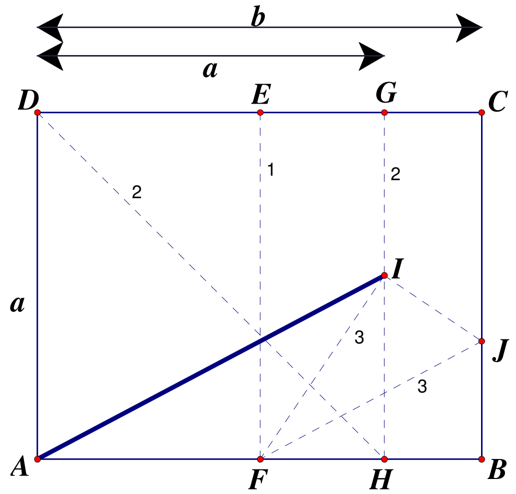
## Given a rectangular paper *ABCD* with $AD = a$, $CD = b$ and $a \le b \le 2a$.

## Using the technique of folding paper to construct $\sqrt{ab}$.

Steps.

1. Fold *AD* to overlap *BC* to get the crease *EF*.
2. Fold *AD* to overlap *GD* (*G* lies on *CD*). The crease is *DH*, *H* lies on *AB*. Join *GH*.
3. Fold *FB* to overlap *FI* (*I* lies on *GH*). The crease is *FJ*, *J* lies on *BC*.

Then $AI = \sqrt{ab}$.




Proof:

By step 1, $AF = FB = \frac{b}{2}$, $EF \perp AB$

By step 2, *AHGD* is a square. $AH = GH = a$, $GH \perp AB$

By step 3, $\Delta FBJ \cong \Delta FIJ$, $FI = FB = \frac{b}{2}$.

$FH = AH - AF = a - \frac{b}{2}$

$IH = \sqrt{FI^2 - FH^2}$ (Pythagoras' theorem)

$= \sqrt{\left(\frac{b}{2}\right)^2 - \left(a - \frac{b}{2}\right)^2} = \sqrt{ab - a^2}$

$AI = \sqrt{AH^2 + HI^2}$ (Pythagoras' theorem)

$= \sqrt{a^2 + ab - a^2}$

$= \sqrt{ab}$

The proof is completed.

Note: *I* must lie between *GH*. $IH \le GH$

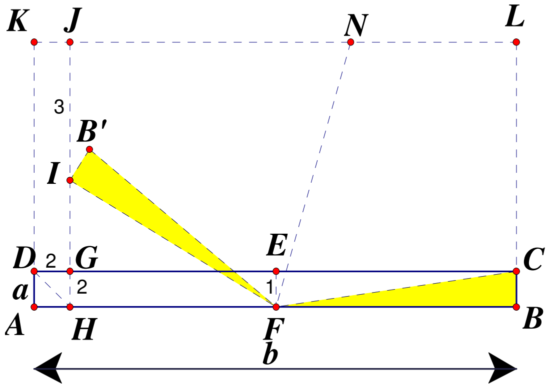
$ab - a^2 \le a^2$

$\therefore b \le 2a$

# If 2*a* < *b*, the following method shows how to construct $\sqrt{ab}$.

Steps.

Put the original paper *ABCD* over a larger piece of rectangular paper *ABLK*, where $\sqrt{ab-b^2} \le AK$.




1. Fold *AD* to overlap *BC* to get the crease *EF*.
2. Fold *AD* to overlap *GD* (*G* lies on *CD*). The crease is *DH*, *H* lies on *AB*. Join *HG* and extend *HG* to meet *LK* at *J*.
3. Fold *FB* to overlap *FI* (*I* lies on *IH*). The crease is *FN*, *N* lies on *KL*.

Then $AI = \sqrt{ab}$.

Proof:

By step 1, $AF = FB = \frac{b}{2}$, $EF \perp AB$

By step 2, *AHGD* is a square. $AH = GH = a$, $GH \perp AB$

By step 3, $\Delta FBC \cong \Delta FIB'$, $FI = FB = \frac{b}{2}$.

$FH = AF - AH = \frac{b}{2} - a$

$IH = \sqrt{FI^2 - FH^2}$ (Pythagoras' theorem)

$= \sqrt{\left(\frac{b}{2}\right)^2 - \left(\frac{b}{2} - a\right)^2} = \sqrt{ab - a^2}$

$AI = \sqrt{AH^2 + HI^2}$ (Pythagoras' theorem)

$= \sqrt{a^2 + ab - a^2}$

$= \sqrt{ab}$

The proof is completed.

Note: *I* must lie between *JH*. $IH \le JH$

$\sqrt{ab - b^2} \le AK$

Note 2: If $b < a$, then we can rename $a$ as $b$ and $b$ as $a$.