Notes on Quadrilaterals

Re-edited by Mr. Francis Hung on 20071219 Last updated: 2021-09-01

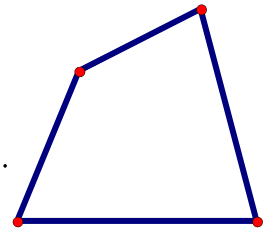
I A **quadrilateral** is a 4-sided polygon.

(a) Interior angle sum is 360°.

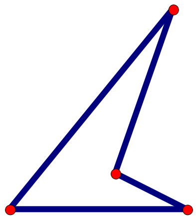
(b) All interior angles of a **convex quadrilateral** < 180°.

(c) Sum of exterior angles of a convex quadrilateral is 360°.

(d) One interior angle of a **concave quadrilateral** > 180°.



convex quadrilateral

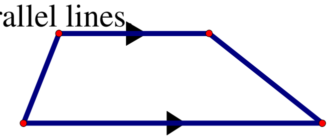


concave quadrilateral

II A **trapezium** is a quadrilateral formed by at least one pair of opposite parallel lines.

(a) It possesses all properties of a quadrilateral.

(b) It possesses all properties of parallel lines.



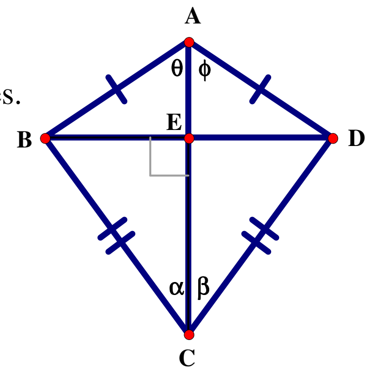
III A **kite** is a convex quadrilateral with two pairs of two equal adjacent sides.

(a) It possesses all properties of a quadrilateral.

(b) One diagonal of a kite bisect the adjacent angles. ($\alpha = \beta$, $\theta = \phi$)

(c) The diagonal in (b) bisects the other diagonal at right angle.

($BE = ED$ and $AC \perp BD$)



IV A **parallelogram** is a quadrilateral formed by two pairs of parallel lines.

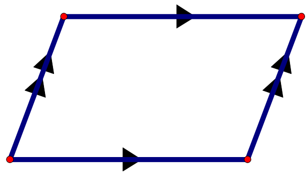
(a) A parallelogram is a trapezium, it has all properties of a trapezium.

(b) The opposite sides of a parallelogram are equal.

(c) The opposite angles of a parallelogram are equal.

(d) The diagonals of a parallelogram bisect each other.

(e) Each diagonal bisect the area of the parallelogram.



V A **rectangle** is a parallelogram with one right angle.

(a) It possesses all properties of a parallelogram.

(b) A rectangle has four right angles.

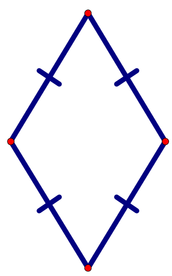
(c) The diagonals of a rectangle are equal.

VI A **rhombus** has four equal sides.

(a) A rhombus is also a kite, it has all properties of a kite.

(b) A rhombus is a parallelogram, it has all properties of a parallelogram.

(c) The diagonals of a rhombus bisect each angle.



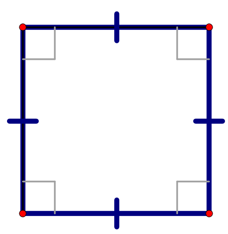
VII A **square** is a rectangle with 2 equal adjacent sides.

(a) A square has four equal sides.

(b) A square is a rhombus, it has all properties of a rhombus.

(c) The diagonals makes 45° with adjacent sides.

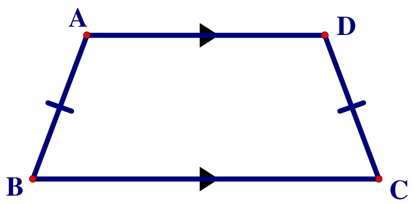
(d) The diagonals are perpendicular to each other.



A. Conditions for a parallelogram:

1. Two pairs of parallel lines.
2. Two pairs of opposite sides are equal.
3. Two pairs of opposite angles are equal.
4. One pair of opposite sides are equal and parallel.
5. Diagonals bisect each other.

Remark: If $AB = CD$ and $AD // BC$, then $ABCD$ may not be a parallelogram.



B. Conditions for a rectangle:

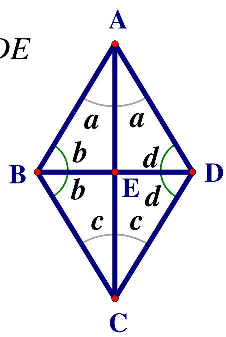
1. Three angles are 90°.
2. It is a parallelogram with two equal diagonals ($AC = BD$).

C. Condition for a rhombus:

1. Four sides equal.
2. The diagonals bisect and perpendicular to each other.
3. The diagonals bisect each angle.

Proof: Given the diagonals AC and BD intersects at E.

Given $\angle BAE=a=\angle DAE, \angle ABE=b=\angle CBE, \angle BCE=c=\angle DCE, \angle ADE = d =\angle CDE$



$2a + b + d = 180°$ ($\angle$ sum of Δ) ⋯⋯(1)

$2b + a + c = 180°$ ($\angle$ sum of Δ) ⋯⋯ (2)

$2c + b + d = 180°$ ($\angle$ sum of Δ) ⋯⋯ (3)

$2d + a + c = 180°$ ($\angle$ sum of Δ) ⋯⋯ (4)

(1) = (3): $a = c$

(2) = (4): $b = d$

$\Delta ABE \cong \Delta CBE \cong \Delta CDE \cong \Delta ADE$ (A.A.S.)

$AB = BC = CD = DA$ (corr. sides, $\cong\Delta$s)

$\therefore$ $ABCD$ is a rhombus

D. Condition for a square:

(1) Four equal sides and one right angle.

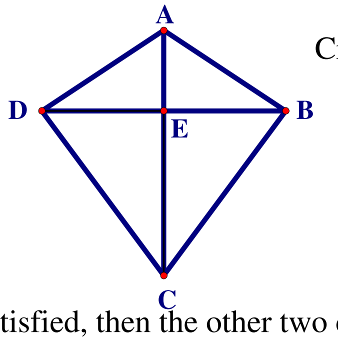
(2) Four right angles and two equal adjacent sides.

(3) The diagonals are equal, perpendicular and bisect each other.

(4) Each diagonal makes 45° with adjacent sides.

E. Theorem of a kite:

Given that AEC and BED are straight lines.



(a) $DE = EB$

(b) $\angle AEB = 90°$

(c) $AD = AB$

(d) $\angle DAC = \angle BAC$, $\angle DCA = \angle BCA$

If any two of the above conditions are satisfied, then the other two conditions are satisfied.

Proof: Suppose (a), (b) are satisfied.

$DE = BE$	Given (a)
$\angle AED = \angle AEB = 90°$	Given (b)
$AE = AE$	common side
$\Delta AED \cong \Delta AEB$	S.A.S.
$AD = AB$	corr. sides, $\cong \Delta$s
$\therefore$ (c) is true.	
$\angle DAE = \angle BAE$	corr. $\angle$s, $\cong \Delta$s
$AC = AC$	common side
$AD = AB$	proved
$\Delta ADC \cong \Delta ABC$	S.A.S.
$\angle DCE = \angle BCE$	corr. $\angle$s, $\cong \Delta$s
$\therefore$(d) is true.	

Suppose (a), (c) are satisfied.

$AE = AE$	Common sides
$DE = BE$	given (a)
$AD = AB$	given (c)
$\Delta AED \cong \Delta AEB$	S.S.S.
$\angle AED = \angle AEB$	corr. $\angle$s, $\cong \Delta$s
$2\angle AEB = 180°$	adj. $\angle$s on st. line
$\angle AEB = 90°$	
(b) is true.	
$\angle DAE = \angle BAE$	corr. $\angle$s, $\cong \Delta$s
$AD = AB$	given (c)
$AC = AC$	common side
$\Delta ACD \cong \Delta ABC$	S.A.S.
$\angle DCA = \angle BCA$	corr. $\angle$s, $\cong \Delta$s
(d) is true.	

Suppose (b), (c) are satisfied.

$AD = AB$	given (c)
$\angle AED = \angle AEB = 90°$	given (b)
$AE = AE$	common side
$\Delta AED \cong \Delta AEB$	R.H.S.
$DE = EB$	corr. sides, $\cong \Delta$s
(a) is true.	
$\angle DAE = \angle BAE$	corr. $\angle$s, $\cong \Delta$s
$AD = AB$	given (c)
$AC = AC$	common side
$\Delta ACD \cong \Delta ABC$	S.A.S.
$\angle DCA = \angle BCA$	corr. $\angle$s, $\cong \Delta$s
(d) is true.	

Suppose (a), (d) are satisfied.

$\angle CAD = \angle CAB$	given (d)
$\angle ACD = \angle ACB$	given (d)
$AC = AC$	common side
$\Delta ACD \cong \Delta ACB$	A.S.A.
$AD = AB$, $CD = CB$	corr. sides, $\cong \Delta$s
(c) is true.	
$DE = EB$	given (a)
$AE = AE$	common side
$\Delta ADE \cong \Delta ABE$	S.S.S.
$\angle AED = \angle AEB$	corr. $\angle$s, $\cong \Delta$s
$2\angle AEB = 180°$	adj. $\angle$s on st. line
$\angle AEB = 90°$	
(b) is true.	

Suppose (b), (d) are satisfied.

$\angle CAD = \angle CAB$	given (d)
$\angle ACD = \angle ACB$	given (d)
$AC = AC$	common side
$\Delta ACD \cong \Delta ACB$	A.S.A.
$AD = AB$, $CD = CB$	corr. sides, $\cong \Delta$s
(c) is true.	
$AE = AE$	common side
$\angle AED = \angle AEB = 90°$	given (b)
$\Delta ADE \cong \Delta ABE$	R.H.S.
$DE = EB$	corr. sides, $\cong \Delta$s
(a) is true.	

Suppose (c), (d) are satisfied.

The proof (a),(b) are left to you as exercise

Summary of Quadrilaterals