Opposite ∠s equal + opposite sides equal does not imply a parallelogram

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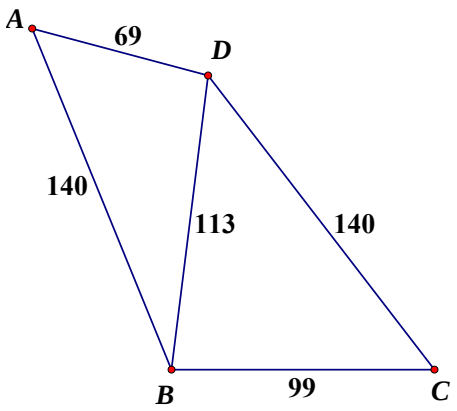
Consider the quadrilateral $ABCD$ with $AB = CD = 140$, $BC = 99$, $AD = 69$, $BD = 113$.



$$\cos A = \frac{140^2 + 69^2 - 113^2}{2 \times 140 \times 69} = \frac{11592}{19320} = \frac{3}{5}$$

$$\cos C = \frac{140^2 + 99^2 - 113^2}{2 \times 140 \times 99} = \frac{16632}{27720} = \frac{3}{5} \Rightarrow \angle A = \angle C$$

$$\cos \angle ADB = \frac{113^2 + 69^2 - 140^2}{2 \times 69 \times 113} = -\frac{2070}{15594} \Rightarrow \angle ADB = 97.6°$$

$$\cos \angle ABD = \frac{113^2 + 140^2 - 69^2}{2 \times 140 \times 113} = \frac{27608}{31640} \Rightarrow \angle ABD = 29.2°$$

$$\cos \angle BDC = \frac{113^2 + 140^2 - 99^2}{2 \times 140 \times 113} = \frac{22568}{31640} \Rightarrow \angle BDC = 44.5°$$

$$\cos \angle CBD = \frac{113^2 + 99^2 - 140^2}{2 \times 99 \times 113} = \frac{2970}{22374} \Rightarrow \angle CBD = 82.4°$$

$\angle ADC = \angle ADB + \angle BDC = 97.6° + 44.5° = 142.1°$

$\angle ABC = \angle ABD + \angle CBD = 29.2° + 82.4° = 111.6°$