Chords of a circle

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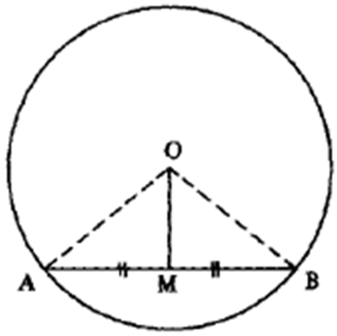
Theorem 1 In the figure, AB is a chord in the circle with centre O, M is the mid-point of AB.

To prove $\angle OMA$ is a right angle.

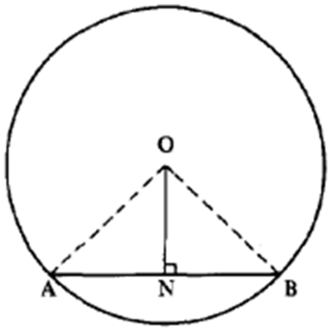


Join OA, OB.

$AM = MB$	(given)
$OA = OB$	(radii)
$OM = OM$	(common sides)
$\Delta OMA \cong \Delta OMB$	(S.S.S.)
$\angle OMA = \angle OMB$	(corr. $\angle$s $\cong$ Δs)
$\angle OMA + \angle OMB = 180°$	(adj. $\angle$s on st. line)
$2\ \angle OMA = 180°$	
$\angle OMA = 90°$	

Abbreviation: line joining centre to mid-point of chord $\perp$ chord

Theorem 2 The straight line drawn from the centre of a circle perpendicular to a chord bisects the chord. (This the converse of theorem 1)



In the figure, AB is a chord in the circle with centre O, $ON \perp AB$.

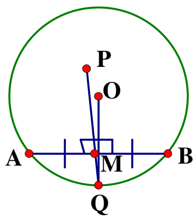
To prove $AN = NB$.

Join OA, OB.

$OA = OB$	(radii)
$OM = OM$	(common sides)
$ON \perp AB$	(given)
$\Delta ONA \cong \Delta ONB$	(R.H.S.)
$AN = NB$	(corr. sides $\cong$ Δs)

Abbreviation: line from centre $\perp$ chord bisects chord

Corollary The perpendicular bisector of a chord of a circle passes through the centre of the circle.



In the figure, AB is a chord in the circle with centre O, PMQ is the perpendicular bisector of AB (M lies on AB). To prove O lies on PQ.

Join OM.

$AM = MB$	(definition of PMB as a $\perp$ bisector of chord AB)
$OM \perp AB$	(line joining centre to mid-point of chord $\perp$ chord)
$PM \perp AB$	(definition of PMB as a $\perp$ bisector of chord AB)
$\therefore PM \parallel OM$	(corr. $\angle$s eq.)

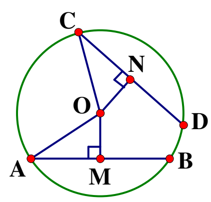
$\because$ Both PM and OM pass through M

$\therefore PM$ and OM overlaps each other.

i.e. O lies on PQ

Abbreviation: $\perp$ bisector of chord passes through centre

Theorem 3 If two chords of a circle are equal, then they are equidistant from the centre



In the figure, $OM \perp AB$, $ON \perp CD$ and $AB = CD$. To prove: $OM = ON$.

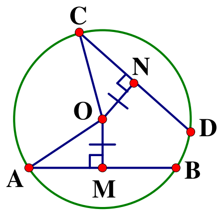
Proof: Join OA, OC.

$OA = OC$	(radii)
$\angle AMO = \angle CNO$	(given)
$AM = \dfrac{1}{2} AB$	(line from centre $\perp$ chord bisects chord)
$= \dfrac{1}{2} CD$	(given)
$= AN$	
$\therefore\ \Delta OAM \cong \Delta OCN$	(R.H.S.)

$OM = ON$ (corr. side $\cong$ Δs)

Abbreviation: **eq. chords are equidistant from centre**

Theorem 4 If two chords of a circle are equidistant from the centre, their lengths are equal



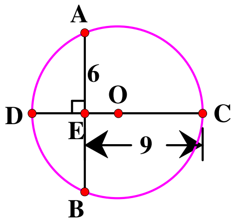
In the figure, $OM \perp AB$, $ON \perp CD$ and $OM = ON$. To prove $AB = CD$.

Proof: Join OA, OC.

$OA = OC$	(radii)
$OM \perp AB$, $ON \perp CD$	(given)
$OM = ON$	(given)
$\Delta OMA \cong \Delta ONC$	(R.H.S.)
$AM = CN$	(corr. sides, $\cong \Delta$s)
$2AM = 2CN$	
$AB = CD$	(line from centre $\perp$ chord bisects chord)

Abbreviation: **chords equidistant from centre are eq**

Example 1



In the figure, CD is the diameter of a circle, centre at O. AB is a perpendicular chord intersecting CD at E. Given that $AE = 6$, $CE = 9$, find the radius of the circle.

Join AD and BC.

$AE = EB = 6$	(line from centre $\perp$ chord bisects chord)
$\angle AED = \angle CEB = 90°$	(vert. opp. $\angle$s)
$\angle EAD = \angle BCE$	($\angle$s in the same segment)
$\angle ADE = \angle CBE$	($\angle$s in the same segment)
$\Delta ADE \sim \Delta CBE$	(equiangular)
$AE : DE = CE : EB$	(corr. sides, $\sim\Delta$s)

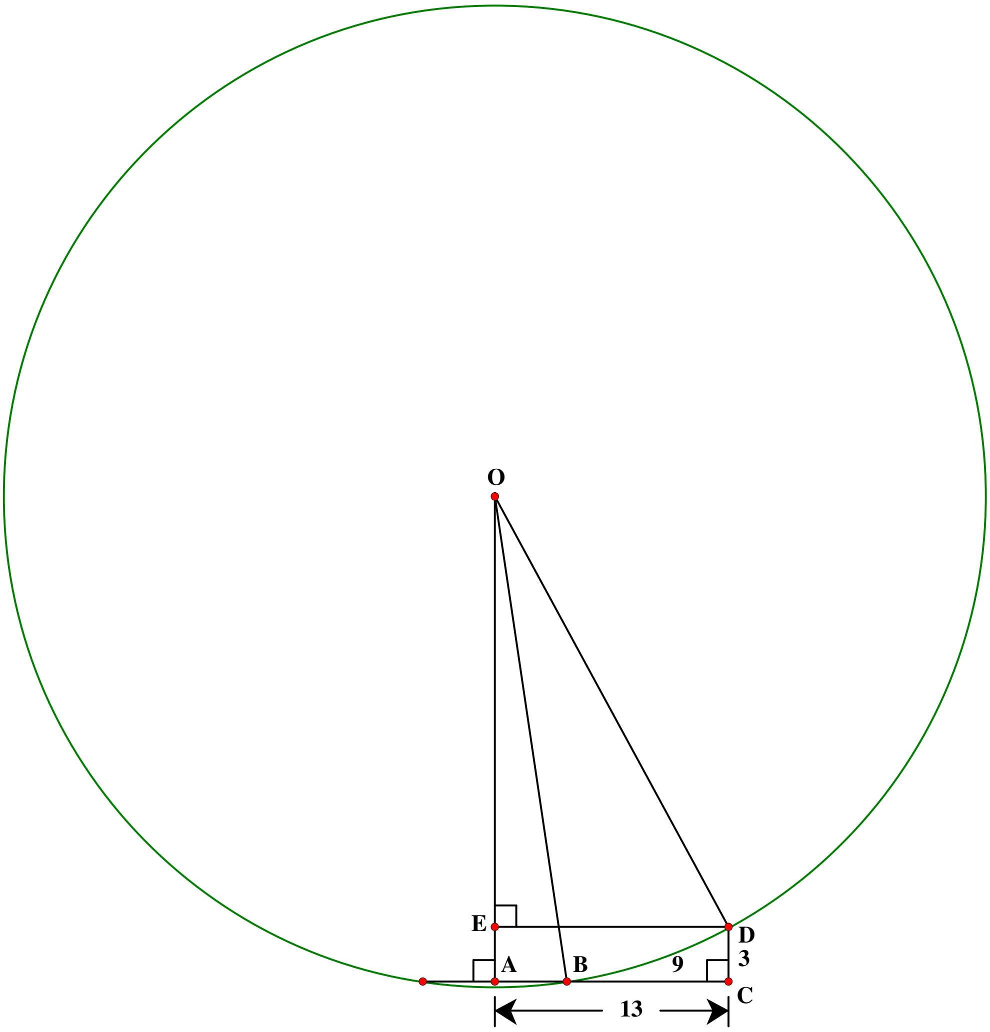
$6 : DE = 9 : 6$

$DE = 4$

$CD = CE + ED = 4 + 9 = 13$

Radius $= CD \div 2 = 6.5$

Example 2



In the figure, ABC, OEA are straight lines. $ACDE$ is a rectangle. B and D are points on a circle centre at O, $AC = 13$, $BC = 9$, $CD = 3$. Find OA and OB.

Solution:

Let $OE = x$, $OB = OD = r$.

$AE = CD = 3$, $DE = CA = 13$ (opp. sides of rectangle)

$OA = x + 3$, $AB = 13 - 9 = 4$

In ΔOAB,

$(x + 3)^2 + 4^2 = r^2 \cdots\cdots (1)$ (Pythagoras' theorem)

In ΔODE,

$x^2 + 13^2 = r^2 \cdots\cdots\cdots (2)$ (Pythagoras' theorem)

$(1) = (2)$: $6x + 9 + 16 = 169$

$x = 24$

Sub. $x = 24$ into (2): $r^2 = 24^2 + 13^2 = 745$

$r = \sqrt{745}$

$\therefore$ $OA = 24 + 3 = 27$

$OB = r = \sqrt{745}$