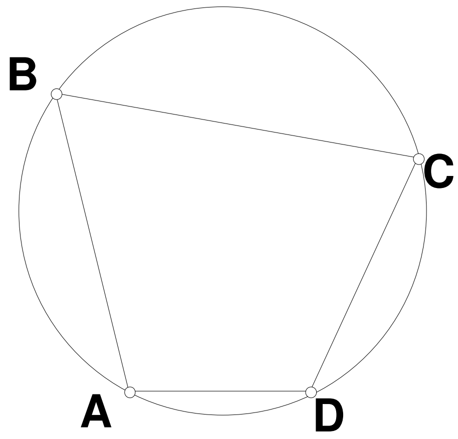
Cyclic quadrilateral

Created by Francis Hung on 20110424 Last updated: 22 September 2021

Theorem The **opposite angles** of a **cyclic quadrilateral** are **supplementary**. **Abbreviation: opp. ∠s cyclic quad.**

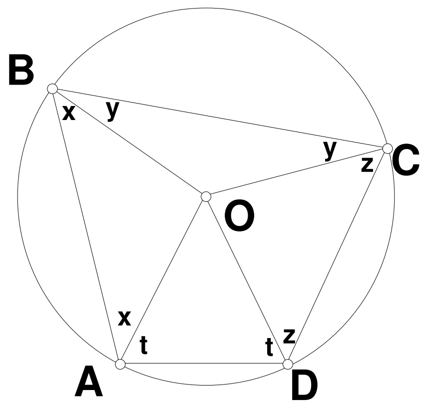


The **theorem** says that a quadrilateral *ABCD* is inscribed in a circle, then

$\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

Proof: Let *O* be the centre of the circle.

Join the radii *OA*, *OB*, *OC* and *OD*.



Let $\angle OAB = \angle OBA = x$ (base ∠s, isos. Δ)

$\angle OBC = \angle OCB = y$ (base ∠s, isos. Δ)

$\angle OCD = \angle ODC = z$ (base ∠s, isos. Δ)

$\angle ODA = \angle OAD = t$ (base ∠s, isos. Δ)

$2x + 2y + 2z + 2t = 360°$ (∠ sum of polygon)

$x + y + z + t = 180°$ …… (1)

$(x + t) + (y + z) = 180°$

$\angle A + \angle C = 180°$

Rearrange (1),

$(x + y) + (t + z) = 180°$

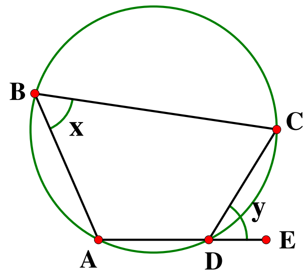
$\angle B + \angle D = 180°$

Hence the **theorem is proved.**

If *AD* is produced to *E*.

Then $\angle CDE$ is called the **exterior angle** of the cyclic quadrilateral.

$\angle ABC$ is the **interior opposite angle** to $\angle CDE$.



Abbreviation:
ext. ∠, cyclic quad.

Theorem $\angle CDE = \angle ABC$ (i.e. $(x = y)$)

Proof: $\angle B + \angle ADC = 180°$ (opp. ∠s, cyclic quad.) …… (2)

$y + \angle ADC = 180°$ (adj. ∠s on st. line) …… (3)

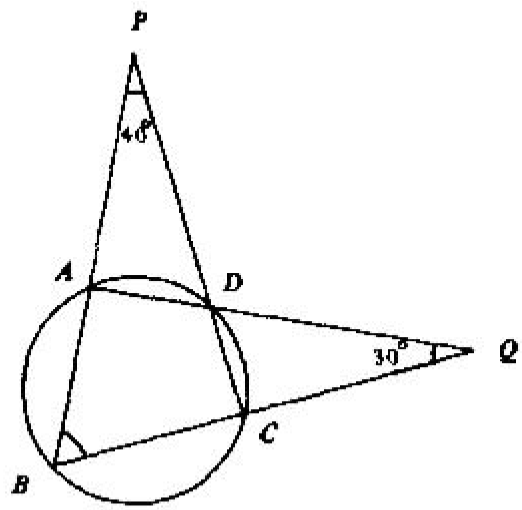
(3) – (2): $x = y$

$\therefore \angle CDE = \angle ABC$

The theorem is proved.

Example 1 (HKCEE 1984 Paper 2 Q54)

In the figure, the chords *BA* and *CD*, when produced, meet at *P*. The chords *AD* and *BC*, when produced, meet at *Q*. Find $\angle B$.



Let $\angle B = x$.

$\angle PAQ = x + 30° = \angle PAD$ (ext. ∠ of Δ)

$\angle BCP = 180° - x - 40°$ (∠ sum of ΔBCP)

$= 140° - x = \angle BCD$

$\angle PAD = \angle BCD$ (ext. ∠, cyclic quad.)

$x + 30° = 140° - x$

$x = 55°$