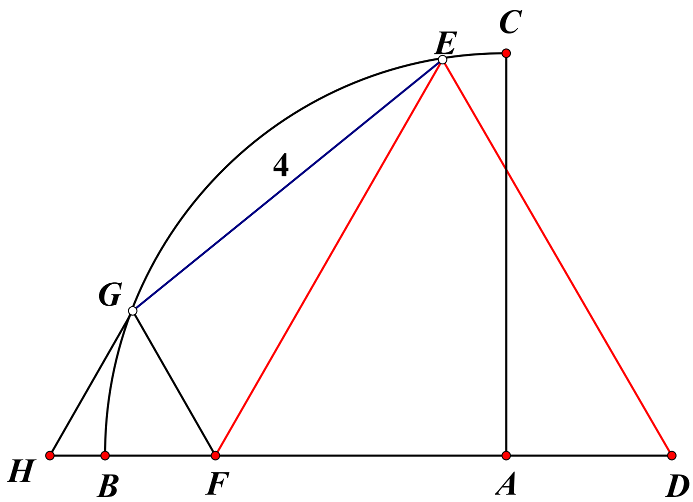
Given ΔDEF and ΔFGH are two equilateral triangles with E, G lying on a quarter of a circle with centre A as shown in the diagram on the right. F lies on AB.
Given that $EG = 4$, what is the area of the quarter of the circle?
(*correct to 3 significant figures*)

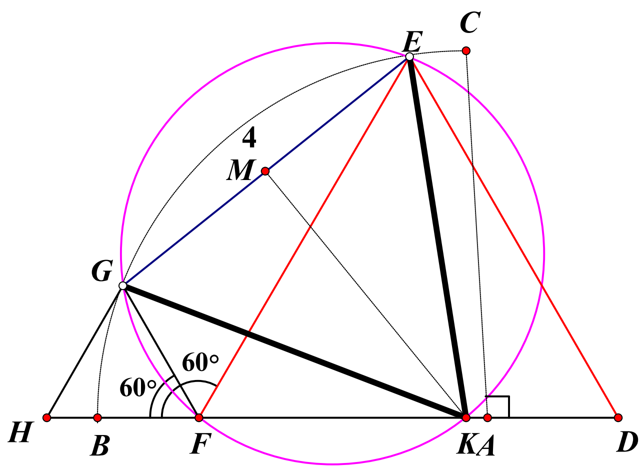
A. 12.6
B. 15.9
C. 18.8
D. 19.6



$\angle GFH = \angle DFE = 60°$ (property of equilateral Δ)
$\angle EFG = 180° - 60° - 60° = 60°$ (adj. $\angle$s on st. line)
Using perpendicular bisectors, construct the circumscribed circle through EFG (pink colour).
Suppose this circle cuts AB at K. Join KE and KG.
$\angle KEG = \angle GFH = 60°$ (ext. $\angle$ cyclic quad.)
$\angle EKG = \angle EFG = 60°$ ($\angle$s in the same segment)
$\angle KGE = 180° - 60° - 60° = 60°$ ($\angle$ sum of ΔEGK)
$\therefore$ ΔEGK is an equilateral Δ
$KE = KG = EG = 4$ (property of equilateral triangle)
Let M be the mid-point of EG. Join KM.
$EM = MG = 2$ (by definition of mid-point)
$KM = KM$ (common side)
$KE = KG = 4$ (proved)
$\therefore$ $\Delta KEM \cong \Delta KGM$ (S.S.S.)
$\angle KME = \angle KMG$ (corr. $\angle$s, $\cong$ Δs)
$\angle KME + \angle KMG = 180°$ (adj. $\angle$s on st. line)
$\therefore$ $\angle KME = \angle KMG = 90°$
$\therefore$ MK is the perpendicular bisector of the chord EG
$\because$ The perpendicualr bisector of a chord must pass through the centre
and the centre A lies on FD
$\therefore$ K and A are the same points
Radius $= AE = KE = 4$

Area of quarter circle $= \frac{1}{4} \cdot \pi \cdot 4^2 = 12.6$

Answer A



Method 2 Provided by Chiu Lut Sau Memorial Secondary School Kwok Po Fai

Let $GF = a$, $FE = DE = b$, $FA = c$, $AG = AE = r$.

$\angle GFE = \angle AFE = \angle EDF = 60°$, $\angle AFG = 120°$

Apply cosine formulas on ΔGFE, ΔAFE, ΔAFG and ΔADE

$4^2 = a^2 + b^2 - ab \cdots\cdots (1)$

$r^2 = a^2 + c^2 + ac \cdots\cdots (2)$

$r^2 = b^2 + c^2 - bc \cdots\cdots (3)$

$r^2 = (b - c)^2 + b^2 - (b - c)b \cdots\cdots (4)$

(2) = (3): $a^2 + c^2 + ac = b^2 + c^2 - bc$

$ac + bc = b^2 - a^2$

$c(a + b) = (b - a)(b + a)$

$c = b - a \cdots\cdots (5)$

Sub. (5) into (4): $r^2 = a^2 + b^2 - ab \cdots\cdots (6)$

(1) = (6): $r = 4$

The area of ther quarter of the circle $= \frac{1}{4} \cdot \pi \cdot 4^2 = 12.6$