Example on concyclic points

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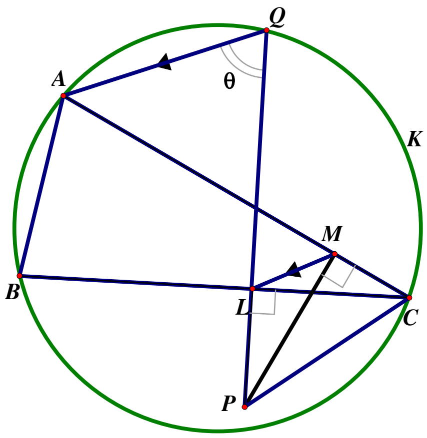
1969 香港中文中學會考高級數學試卷二 Q10

(i) 圓(K)為 ΔABC 之外接圓。P 為 ΔABC 外的一點。 L、M 為從 P 至 BC，AC 兩邊之垂足。若 PL 之延綫交 (K) 於 Q 且 AQ 平行於 ML，試證 P 點必在圓 (K) 上。
設 N 為從 P 至 BA 成 BA 之延綫垂足，試證 L、M、N 三點共綫。

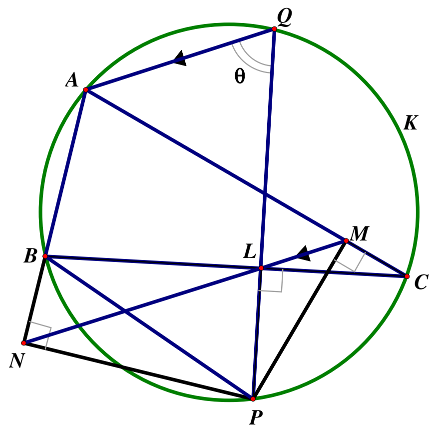
(ii) 若ΔABC 之三高 AD、BE、CF 相交於 H，$DS \perp BE$，$DT \perp CF$，$DX \perp BE$，$DY \perp AB$。證明 S、T、X、Y 四點共綫。

(i) Let $\angle AQP = \theta$



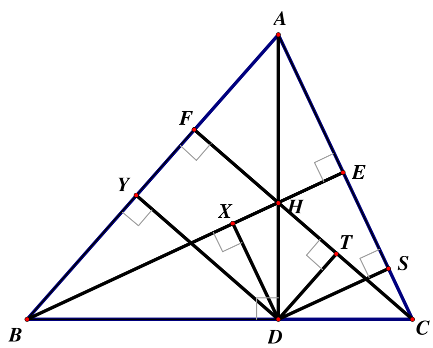
$\angle QLM = \theta$	alt. $\angle$s, AQ // LM
$\angle CLM = 90° - \theta$	adj. $\angle$s on st. line
$\angle PLM = 180° - \theta$	adj. $\angle$s on st. line
$\because \angle CLP = 90° = \angle CMP$	by construction
C, M, L, P are concyclic	converse, $\angle$s in the same seg
$\angle PCM = 180° - \angle PLM$	opp. $\angle$, cyclic quad.
$= \theta$	
$\therefore \angle AQP = \theta = \angle ACP$	
A, Q, C, P are concyclic	converse, $\angle$s in the same seg
$\therefore$ P lies on the circle.	

Join BP and NL.



$\angle NBP = \theta$	ext. $\angle$ of cyclic quad.
$\because \angle BNP = 90° = \angle PLC$	by construction
$\therefore B$, N, P, L are concyclic	ext $\angle$ = int. opp. $\angle$
$\angle NLP = \theta$	$\angle$s in the same seg.
$\therefore \angle AQP = \theta = \angle NBP$	
AQ // NL	corr. $\angle$s eq.
$\because$ AQ // LM and AQ // NL	
$\therefore$ NL // LM	transitive property of parallel lines
$\therefore$ N, L, M are collinear.	

(ii)



$\angle CEH + \angle CDH = 180°$	given
$\therefore C$, D, H, E are concyclic	opp. $\angle$ supp.
Apply the result of (i) to ΔCEH with $P = D$.	
X, T, S are collinear.	
$\angle BFH + \angle BDH = 180°$	given
$\therefore$ B, D, H, F are concyclic	opp. $\angle$ supp.
Apply the result of (i) to ΔBFH with $P = D$.	
Y, X, T are collinear.	
$\therefore$ S, T, X, Y are collinear.	