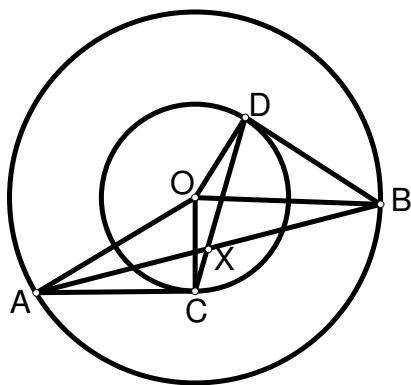


Concyclic points Example 3

1972 General Mathematics Syllabus 1 (Chinese) Paper 2 Q10

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The figure shows two concentric circles with a common centre O . A and B are two points on the outer circle. AC and BD touch the inner circle at C and D respectively. AB and CD intersect at X . Prove that

- (a) $AC = BD$.
 - (b) $\triangle OAB \sim \triangle OCD$.
 - (c) CD bisects AB at X .
- (a) $OC = OD$; $OA = OB$ (same radii)
 $\angle OCA = \angle ODB = 90^\circ$ (tangent \perp radius)
 $\triangle OAC \cong \triangle OBD$ (R.H.S.)
 $AC = BD$ (corr. sides \cong Δ s)
- (b) $OA : OC = OB : OD$ (same radii)
 $\angle AOB = \angle AOC + \angle COB = \angle COB + \angle BOD$ (corr. \angle s \cong Δ s)
 $= \angle COD$
 $\triangle OAB \sim \triangle OCD$ (2 sides proportional, 1 included \angle , S.A.S.)
- (c) $\angle OBA = \angle ODC$ (corr. \angle s, $\sim \Delta$ s)
 O, X, B, D are concyclic. (converse, \angle s in the same segment)
 $\angle OXB + \angle ODB = 180^\circ$ (opp. \angle s, cyclic quad.)
 $\angle OXB = 90^\circ$
 $AX = XB$ (\perp from centre bisect chord)
 CD bisects AB at X .