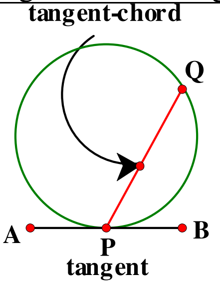
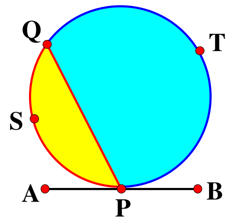
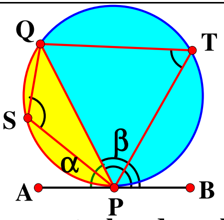
# Angle in alternate segment

Created by Mr. Francis Hung on 20210926 Last updated: 2021-09-26

1. Tangent-chord, tangent-chord segments, angles in tangent-chord segment.

<table>
<tr>
<td>

AB is a tangent to the circle at P. PQ is a chord, called the tangent chord.</td>
<td>

The tangent-chord PQ divides the circle into two segments $\overset{\frown}{PSQ}$ and $\overset{\frown}{PTQ}$ called tangent-chord segments.</td>
<td>

Tangent-chord angles<br>
Tangent-chord angles refer to $\angle APQ = \alpha$ or $\angle BPQ = \beta$.</td>
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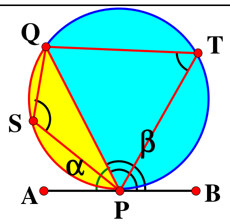
<td>
</td>
<td>Segment $\overset{\frown}{PTQ}$ lies on the opposite side of $\angle APQ$, Hence, $\angle PTQ$ is called an angle in alternate segment corresponding to $\angle APQ = \alpha$.</td>
<td>Similarly, segment $\overset{\frown}{PSQ}$ lies on the opposite side of $\angle BPQ$, Hence, $\angle PSQ$ is called an angle in alternate segment corresponding to $\angle BPQ = \beta$.</td>
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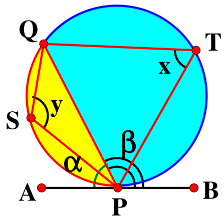
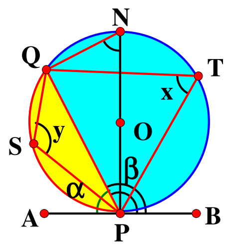
**Theorem 1** A tangent-chord angle of a circle is equal to an angle in the alternate segment.




In the figure, $AB$ is a tangent at $P$. To prove $\alpha = x$, $\beta = y$ .

Abbreviation: **$\angle$ in alt. segment**

Proof: Let the centre be $O$.

Draw the diameter $PON$.

Join $QN$.

$\angle PNQ = \angle PTQ = x$ ($\angle$s in the same segment)

$\angle PQN = 90°$ ($\angle$ in semi-circle)

In $\Delta PQN$,

$\angle NPQ = 180° - 90° - x = 90° - x$ ($\angle$ sum of $\Delta$)

$\alpha + \angle NPQ = 90°$ (tangent $\perp$ radius)

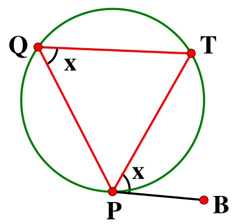
$\alpha = x$

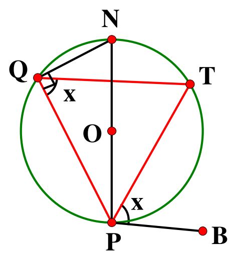
$x + y = 180°$ (opp. $\angle$s, cyclic quad.)

$\alpha + \beta = 180°$ (adj. $\angle$s on st. line)

$\therefore \beta = y$

**Theorem 2** In the figure, given the circle $PQT$. $\angle BPT = \angle PQT = x$. To prove $BP$ is a tangent at $P$.




Abbreviation: **Converse, $\angle$ in alt. segment**

Proof: Let the centre be $O$.

Draw the diameter $PON$.

Join $QN$.

$\angle PQN = 90°$ ($\angle$ in semi-circle)

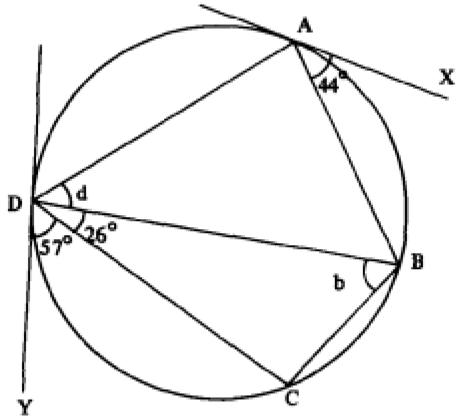
$\angle NQT = 90° - x$

$\angle NPT = \angle NQT = 90° - x$ ($\angle$s in the same segment)

$\angle NPT + \angle BPT = 90° - x + x = 90°$

$\therefore$ $PB$ is a tangent at $P$ (converse, tangent $\perp$ radius)

**Example 1**




In the figure, $ABCD$ is a cyclic quadrilateral. $AX$ and $DY$ are tangents, $\angle XAB = 44^\circ$, $\angle YDC = 57^\circ$, $\angle BDC = 26^\circ$. To find the angles of quadrilateral $ABCD$.

Using the notation in the figure,

$b = 57^\circ$ ($\angle$ in alt. segment)

$d = 44^\circ$ ($\angle$ in alt. segment)

$\angle C + 26^\circ + 57^\circ = 180^\circ$ ($\angle$ sum of $\Delta BCD$)

$\angle C = 97^\circ$

$\angle A + \angle C = 180^\circ$ (opp. $\angle$s, cyclic quad.)

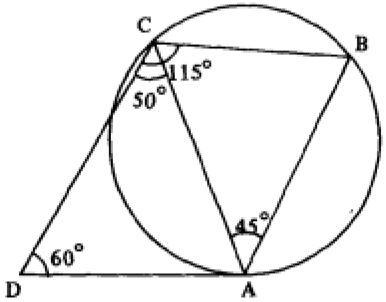
$\angle A = 180^\circ - 97^\circ = 83^\circ$

$\angle ADC = d + 26^\circ = 44^\circ + 26^\circ = 70^\circ$

$\angle ABC + \angle ADC = 180^\circ$ (opp. $\angle$s, cyclic quad.)

$\angle ABC = 180^\circ - 70^\circ = 110^\circ$

**Example 2**




As shown in the figure, To prove: $DA$ tocues the circle $ABC$ at $A$.

$\angle ACB = 115^\circ - 50^\circ = 65^\circ$

$\angle B + 65^\circ + 45^\circ = 180^\circ$ ($\angle$ sum of $\Delta ABC$)

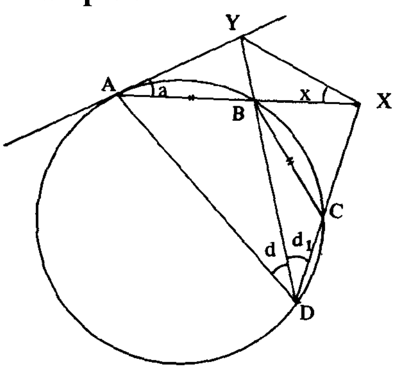
$\angle B = 70^\circ$

$\angle CAD = 180^\circ - 60^\circ - 50^\circ = 70^\circ$ ($\angle$ sum of $\Delta ACD$)

$\because \angle B = 70^\circ = \angle CAD$

$\therefore$ $DA$ is a tangent to the circle $ABC$ at $A$ (converse, $\angle$ in alt. seg.)

**Example 3**




In the figure, $ABCD$ is a minor arc of a circle, such that $AB = BC$, $AB$ and $DC$ are produced to meet at $X$ and $DB$ is produced to meet the tangent $AY$ at $Y$. To prove: $YX = YA$.

Join $AD$. Using the notation in the figure,

$d = a$ ($\angle$ in alt. segment)

$d = d_1$ (eq. chords eq. $\angle$s)

$\therefore a = d_1$

$A$, $D$, $X$, $Y$ are concyclic (converse, $\angle$s in the same seg.)

$x = d$ ($\angle$s in the same segment)

$\therefore a = x$

$YX = YA$ (sides opp. eq. $\angle$s)