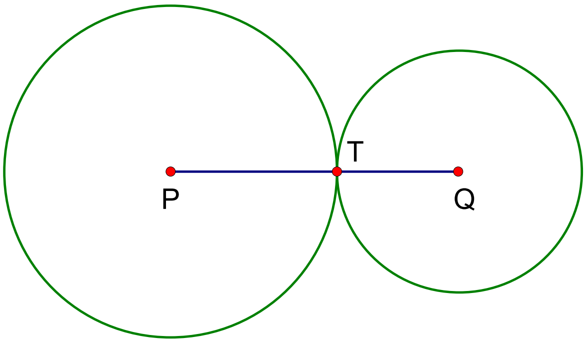
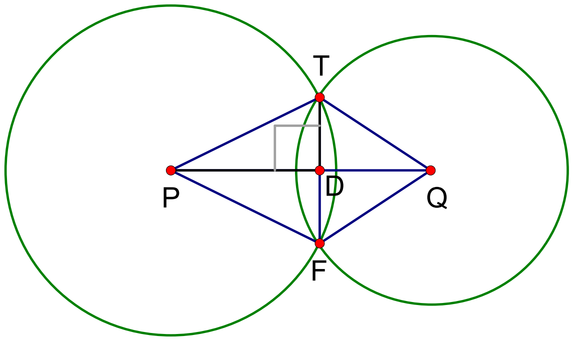
Common tangent

Created by Francis Hung on 20101016 Last updated: 21 September 2021

Two circles with centres P, Q touches each other at T. **To prove that P, T, Q are collinear.**





Suppose P, T, Q are **not collinear**. Join PQ. A triangle PTQ is formed.

Let D be the foot of perpendicular drawn from T onto PQ.

Produce TD to F so that $TD = DF$. Join PF, QF.

$PD = PD$	common sides
$TD = DF$	by construction
$\angle PDT = \angle PDF = 90°$	by construction
$\therefore \Delta PDT \cong \Delta PDF$	S.A.S.
$PT = PF$	corr. sides $\cong$ Δs
$QD = QD$	common sides
$TD = DF$	by construction
$\angle QDT = \angle QDF = 90°$	by construction
$\therefore \Delta QDT \cong \Delta QDF$	S.A.S.
$QT = QF$	corr. sides $\cong$ Δs

$\because PT = PF =$ radii and $QT = QF =$ radii

$\therefore F$ lies on the two circles.

$\Rightarrow$ The two circles intersect at two distinct points T, F .

This contradicts to the fact that they touch each other at only one point T .

Therefore, P, T, Q are collinear.

If a line GH is drawn through T and perpendicular to PQ, then it is a common tangent to both circles.

(converse, tangent $\perp$ radius).