Tangent from external point Example

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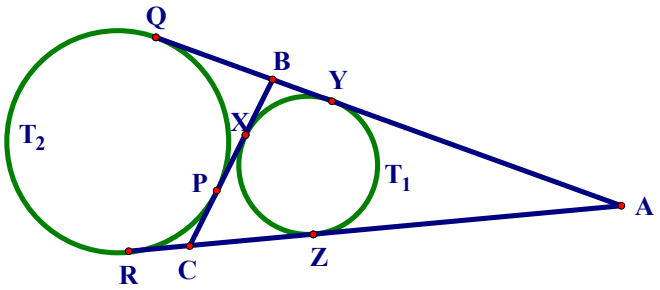
1961 HKU O Level Pure Mathematics Paper 2 Q2(a)

In the figure, the circle T_1 is the **in-circle** and T_2 is the **ex-circle** of the triangle ABC and the points X, Y, Z, P, Q, R are the **points of contact**.



Prove that (a) $AQ = AR = s$

(b) $AY = AZ = s - a$

(c) $YQ = RZ = a$, where $2s$ is the perimeter of the triangle ABC and $a = BC$.

(a) $AQ = AR$ (tangent from ext. point)

$BP = BQ$ (tangent from ext. point)

$CP = CR$ (tangent from ext. point)

$AQ + AR = AB + (BP + CP) + AC$

$= AB + BC + AC$ = perimeter of ΔABC

$= 2s$

$\therefore AQ = AR = s$

(b) $AY = AZ$ (tangent from ext. point)

$BX = BY$ (tangent from ext. point)

$CX = CZ$ (tangent from ext. point)

$AY + AZ = AB - BY + AC - CZ$

$= AB + AC + BC - BC - (BX + CX)$

$= 2s - 2a$

$\therefore AY = AZ = s - a$

(c) $YQ = AQ - AY$

$= AR - AZ$ (by (a) and (b))

$= RZ$

$YQ = AQ - AY = s - (s - a) = a$

$\therefore YQ = RZ = a$