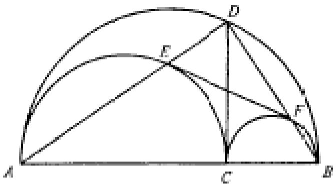
Tangent Problem 3

1974 Mathematics (Alternative syllabus A) Paper 2 Section B Q9

Created by Francis Hung Last updated: 03 September 2021

In the figure, AC, CB and AB are the diameters of semi-circles AEC, CFB and ADB respectively. $CD \perp AB$. AED and BFD are straight lines.



(a) Prove that

(i) $EF = DC$

(ii) EF is the common tangent of the circles AEC and CFB .

(b) If $AC = 9$ cm, $CB = 4$ cm, find EF .

(a) (i) $\angle AEC = \angle CFB = \angle ADB = 90°$ ($\angle$ in semi-circle)

$\angle ECF = 90°$ ($\angle$ sum of polygon)

$CEDF$ is a rectangle.

$CD = EF$ (diagonals of rectangle)

(ii) It can be easily proved that $\Delta EFC \cong \Delta CDE$ (S.S.S.)

Let $\angle CEF = \angle DCE = \theta$

$\because CD \perp AB$ (given)

$\therefore \angle ACE = 90° - \theta$

$\angle EAC = 90° - (90° - \theta) = \theta = \angle CEF$

$\therefore$ EF is a tangent touching the circle AEC at E. (converse, $\angle$ in alt. segment)

Similarly, EF is also a tangent touching the circle CFB at F.

(b) $AC \times CB = CD^2$ (intersecting chords theorem)

$CD^2 = 9 \text{ cm} \times 4 \text{ cm} = 36 \text{ cm}^2$

$CD = 6$ cm

$EF = 6$ cm