## **Tangent Problem 4**

## **HKCEE Mathematics Paper 2 1992 Q50**

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 $\boldsymbol{A}$ 

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B

In the figure, the two circles touch each other at C. The diameter AB of the bigger circle is tangent to the smaller circle at D. If DE bisects  $\angle ADC$ , find  $\theta$ .



B. 38°

C. 45°

D. 52°

E. 66°

## Method 1

Let 
$$\angle ADE = x = \angle CDE$$

$$\angle ACD = x$$

$$\angle ACB = 90^{\circ} = x + \theta \cdots (1) \ (\angle \text{ in semi-circle})$$

 $(\angle in alt. seg.)$ 

$$\angle ABC = 66^{\circ}$$
 ( $\angle$  sum of  $\triangle ABC$ )

$$66^{\circ} + \theta = 2x \cdot \cdots \cdot (2)$$
 (ext.  $\angle$  of  $\triangle BCD$ )

$$2(1) - (2) 114^{\circ} - \theta = 2\theta$$

 $\theta = 38^{\circ}$ , the answer is B.

## Method 2

Let *O* be the centre of the bigger circle.

Let *P* be the centre of the smaller circle.

Then O, P, C are collinear (why?)

Join *PD*. Then  $PD \perp AB$  (tangent  $\perp$  radius)

$$\angle BOC = 48^{\circ} \ (\angle \text{ at centre twice } \angle \text{ at } \odot^{\text{ce}})$$

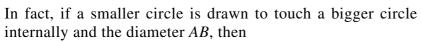
$$\angle CPD = 48^{\circ} + 90^{\circ} \text{ (ext. } \angle \text{ of } \Delta CDP)$$
  
= 138°

CP = DP = radii of the smaller circle

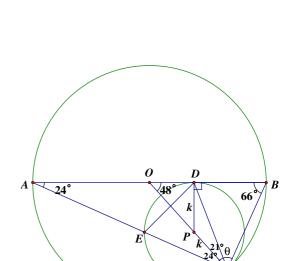
$$\angle PCD = \angle PDC = \frac{180^{\circ} - 138^{\circ}}{2}$$
 (base  $\angle$ s, isos.  $\triangle$ )

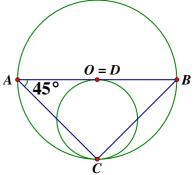
$$\angle OBC = \angle OCB = 66^{\circ}$$
 (base  $\angle$ s, isos.  $\Delta$ )

$$21^{\circ} + \theta = 66^{\circ} \Rightarrow \theta = 45^{\circ}$$
, the answer is C.



$$\angle BAC = 45^{\circ} \neq 24^{\circ}$$
 and  $O = D$ 





On the other hand, if  $\angle BAC = 24^{\circ}$  and a smaller circle is drawn to touch the diameter AB of the bigger circle, then the smaller circle will not touch the bigger circle at C. It will intersect the bigger circle at two distinct points.

This question is wrong.

