

Tangent Problem 4

HKCEE Mathematics Paper 2 1992 Q50

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In the figure, the two circles touch each other at C . The diameter AB of the bigger circle is tangent to the smaller circle at D . If DE bisects $\angle ADC$, find θ .

- A. 24°
- B. 38°
- C. 45°
- D. 52°
- E. 66°

Method 1

Let $\angle ADE = x = \angle CDE$

$\angle ACD = x$ (\angle in alt. seg.)

$\angle ACB = 90^\circ = x + \theta$ (1) (\angle in semi-circle)

$\angle ABC = 66^\circ$ (\angle sum of $\triangle ABC$)

$66^\circ + \theta = 2x$ (2) (ext. \angle of $\triangle BCD$)

$$2(1) - (2) \quad 114^\circ - \theta = 2\theta$$

$\theta = 38^\circ$, the answer is B.

Method 2

Let O be the centre of the bigger circle.

Let P be the centre of the smaller circle.

Then O, P, C are collinear (why?)

Join PD . Then $PD \perp AB$ (tangent \perp radius)

$\angle BOC = 48^\circ$ (\angle at centre twice \angle at \odot^{ce})

$\angle CPD = 48^\circ + 90^\circ$ (ext. \angle of $\triangle CDP$)
 $= 138^\circ$

$CP = DP =$ radii of the smaller circle

$$\angle PCD = \angle PDC = \frac{180^\circ - 138^\circ}{2} \quad (\text{base } \angle\text{s, isos. } \triangle)$$

$$= 21^\circ$$

$\angle OBC = \angle OCB = 66^\circ$ (base $\angle\text{s, isos. } \triangle$)

$21^\circ + \theta = 66^\circ \Rightarrow \theta = 45^\circ$, the answer is C.

In fact, if a smaller circle is drawn to touch a bigger circle internally and the diameter AB , then

$\angle BAC = 45^\circ \neq 24^\circ$ and $O = D$

On the other hand, if $\angle BAC = 24^\circ$ and a smaller circle is drawn to touch the diameter AB of the bigger circle, then the smaller circle will not touch the bigger circle at C . It will intersect the bigger circle at two distinct points.

This question is wrong.

