Tangent Problem 5

American High School Mathematics Examination 1965 Q39

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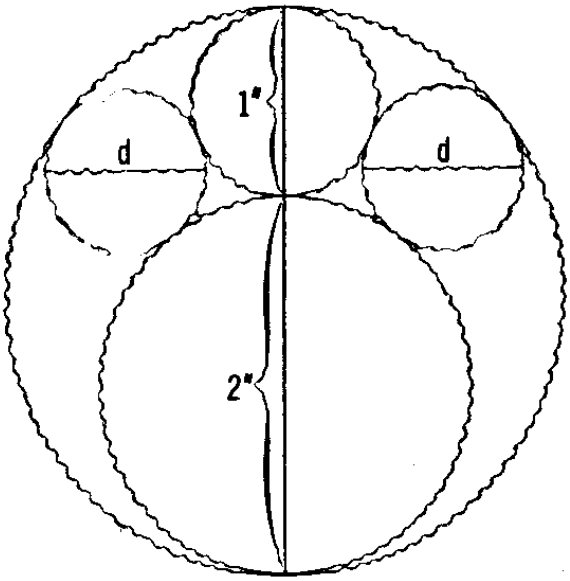
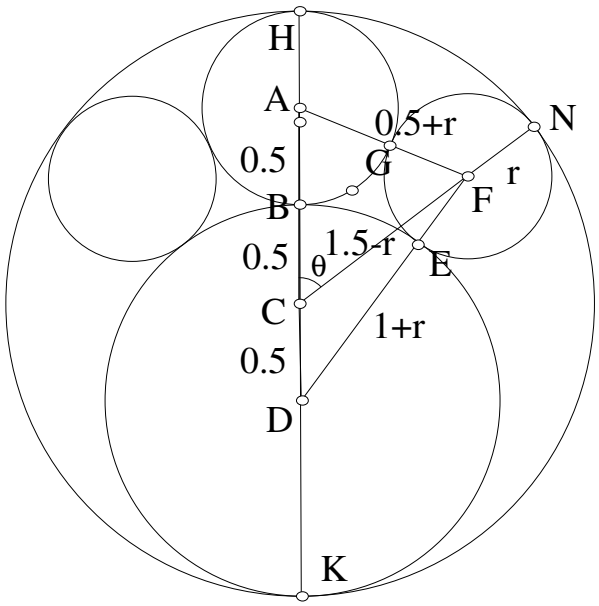
A foreman noticed an inspector checking a 3 cm hole with a 2 cm plug and a 1 cm plug and suggested that two more gauges be inserted to be sure that the fit was snug. If the new gauges are alike, find the diameter, *d* cm.





Let *A* be the centre of the 1 cm plug, *D* be the 2 cm plug, *C* be the 3 cm hole. Then *A*, *C*, *D* are collinear. Join *A*, *C* and *D* and produce the line segment to two ends of the diameter *HK*.

Let *B* be the intersection of the 2 cm plug and 3 cm hole, *F* be the centre of the right gauge with radius *r* cm as shown. Let *E* be the intersection of the 2 cm plug and the right gauge *F*, *G* be the intersection of the 1 cm plug and the right gauge *F*. Then *A*, *G*, *F* are collinear; *D*, *E*, *F* are collinear.

Produce *CF* to the intersection (*N*) of the 3 cm hole and the right gauge.

AB = 0.5 cm (radius of the 1 cm plug); *BD* = 1 cm (radius of the 2 cm plug)

HK = *HB* + *BK* = 1 cm + 2 cm = 3 cm

CH = 1.5 cm = *CN* (radius of the 3 cm hole)

CB = *CH* – *BH* = 1.5 cm – 1 cm = 0.5 cm

CD = *BD* – *BC* = 1 cm – 0.5 cm = 0.5 cm

CN = 1.5 cm (radius of the 3 cm hole); *FN* = *r* cm (radius of the right gauge)

CF = *CN* – *FN* = (1.5 – *r*) cm; *AF* = *AG* + *GF* = (0.5 + *r*) cm; *DF* = *DE* + *EF* = (1 + *r*) cm

Let $\angle ACF = \theta$, then $\angle DCF = 180° - \theta$ (adj. $\angle$s on st. line)

Apply cosine rule on ΔACF: $\cos\theta = \dfrac{1+(1.5-r)^2-(0.5+r)^2}{2(1)(1.5-r)}$ ……(1)

Apply cosine rule on ΔDCF: $\cos(180° - \theta) = \dfrac{0.5^2+(1.5-r)^2-(1+r)^2}{2(0.5)(1.5-r)}$ …… (2)

(1) + (2) $0 = \dfrac{1+2.25-3r+r^2-0.25-r-r^2}{2(1.5-r)} + \dfrac{0.25+2.25-3r+r^2-1-2r-r^2}{1.5-r}$

$(3 - 4r) + 2(1.5 - 5r) = 0$

$r = \dfrac{3}{7} \Rightarrow d = \dfrac{6}{7}$