Given ΔABC. To construct 3 circles touching each other externally at A, B, C

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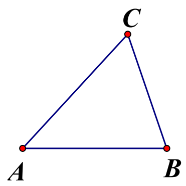

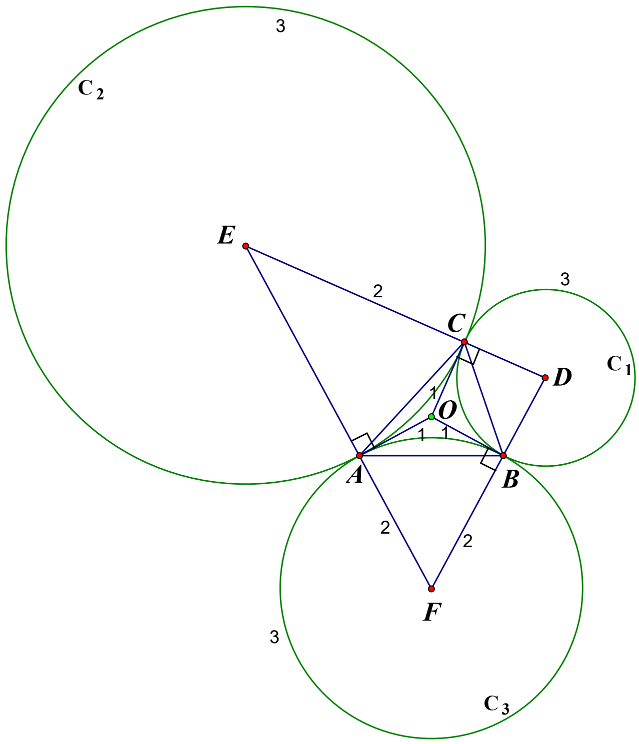

Steps

(1) Using perpendicular bisectors to construct the circumcentre O.

https://twhung78.github.io/Geometry/7%20Construction%20by%20ruler%20and%20compasses/circle/Circumscribed_circle.pdf

(2) Draw $EAF \perp OA$, $DBF \perp OB$, $DCE \perp OC$.

(3) Use D as centre, radius = DB to draw a circle C_1, use E as centre, radius = EC to draw a circle C_2, use F as centre, radius = FA to draw a circle C_3.

Then the 3 circles touch each other externally at A, B and C respectively.

Proof: $\because$ O is circumcentre of ABC

$\therefore OA = OB = OC$	(circumradius)
$\angle OAB = \angle OBA$, $\angle OBC = \angle OCB$, $\angle OAC = \angle OCA$	(base $\angle$s isosceles Δ)
$\angle FAB = \angle FBA$, $\angle EAC = \angle ECA$, $\angle DBC = \angle DCB$	($EAF \perp OA$, $DBE \perp OB$, $DCF \perp OC$)
$FA = FB$, $EA = EC$, $DB = DC$	(sides, opp. eq. $\angle$s)
C_1 pass through B, C; C_2 pass through A, C, C_3 pass through A, B	
By construction, $EAF \perp OA$, $DBF \perp OB$, $DCE \perp OC$.	
$\therefore OA$ is a common tangent to C_2, C_3 at A	(converse, tangent $\perp$ radius)
OB is a common tangent to C_1, C_3 at B	(converse, tangent $\perp$ radius)
OC is a common tangent to C_1, C_2 at C	(converse, tangent $\perp$ radius)
$\therefore$ The 3 circles touches each other externally at A, B and C respectively.	