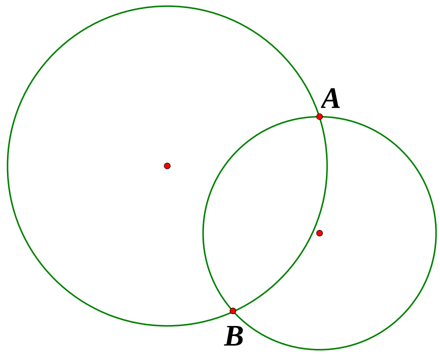
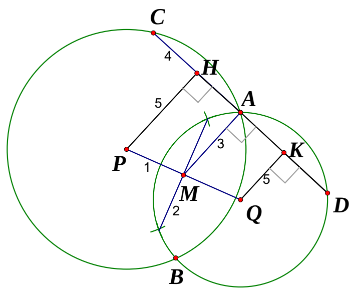
二不等圓交於 A，B。求過 A 作一直綫分別交此二圓於 C 與 D，使 $CA = AD$。

1968 香港中文中學會考普通數學 Paper 2 Q16 (a)

Created by Mr. Francis Hung on 20160928 Last updated: 2023-07-03





Let the centres of the two circles be P and Q respectively.

(1) Join PQ.

(2) Draw the perpendicular bisector of PQ. M is the mid-point.

(3) Join MA.

(4) Through A draw a chord perpendicular to MA, cutting the two circles at C and D respectively.

(5) Draw $PH \perp CD$, $QK \perp CD$. H and K are the feet of perpendiculars.

Then CAD is the required chord.

Proof: $PH \parallel MK \parallel QK$ (corr. ∠s eq.)

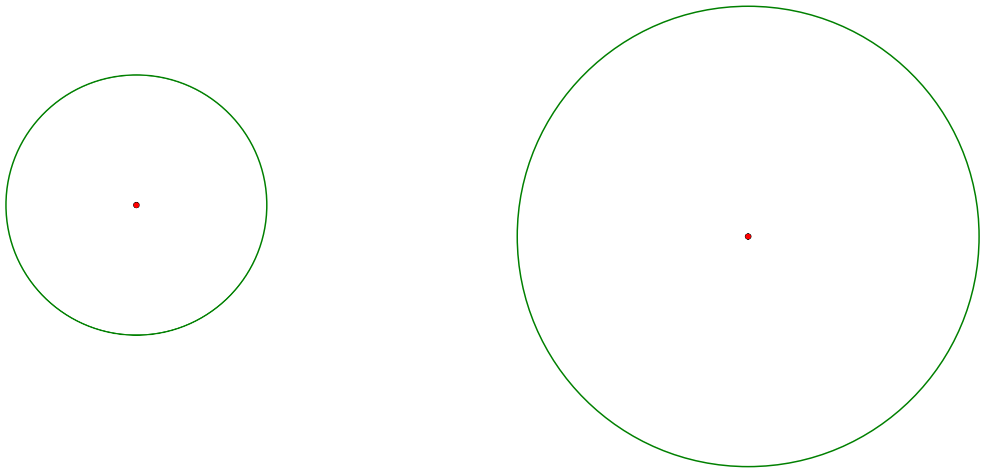
$PM = MQ$ (by construction)

$AH = AK$ (intercept theorem)

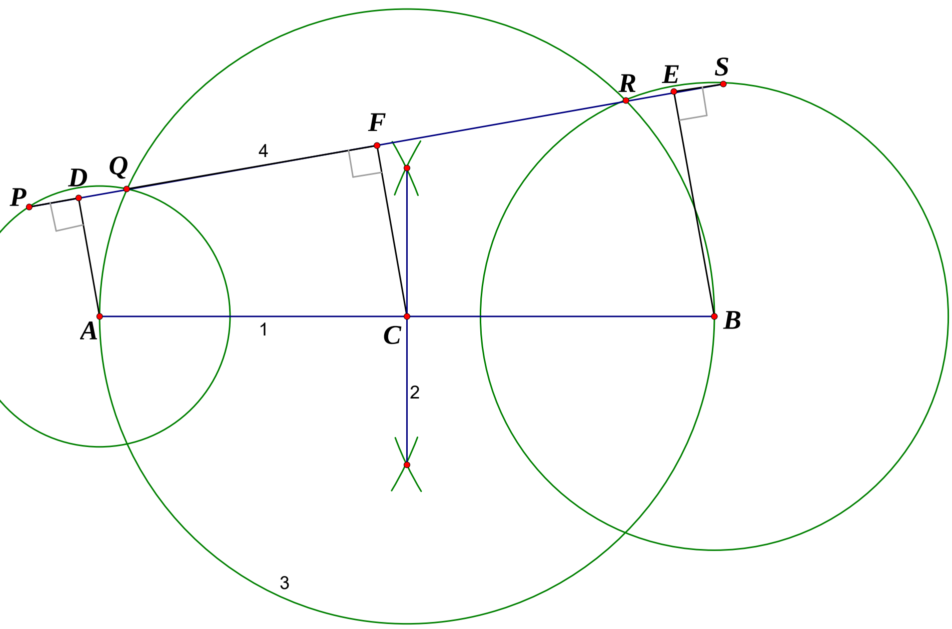
$CH = HA$ and $AK = KD$ (⊥ from centre bisect chord)

$\therefore AC = 2AH = 2AK = AD$

二不等圓不相交。求作一直綫分別交此二圓於 P、Q、R 及 S，使 $PQ = RS$。



Construction steps:



Let A and B be the centres of the two non-intersecting circles.

(1) Join A, B.

(2) Draw the perpendicular bisector of AB, let C be the mid-point of AB.

(3) Use C as centre, CA as radius to draw a circle, intersecting the original circles at Q, R.

(4) Join QR and extend the line both ways, cutting the 2 circles again at P and S as shown.

Then $PQ = RS$.

Proof: Draw $AD \perp PQ$, $CF \perp QR$, $BE \perp RS$ as shown.

$PD = DQ$, $QF = FR$, $RE = ES$ (⊥ from centre bisects chord)

$AD \parallel CF \parallel BE$ (corr. ∠s eq.)

$DF = FE$ (intercept theorem)

$DQ = DF - QF = FE - FR = RE$

$2\,DQ = 2\,RE$

$PQ = RS$ (⊥ from centre bisects chord)

Remark: The circle in step 3 does not necessary pass through A and B.

The only requirement is cutting the two circles.