已給 P 點在一圓形內，圓心為 C，另給一已知綫段 MN。過 P 作一弦綫 QR，使得 $QR = MN$。

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Figure 3 shows a circle of centre C and a line segment MN. P is a point lies inside the circle. Construct a chord QR with points Q and R on the circumference of the circle so that it passes through P and its length is equal to that of MN.

圖三所示為一以 C 為圓心的圓及一綫段 MN。P 為該圓內的一點。試作一通過 P 的弦綫 QR，其中 Q 及 R 為圓周上的點，且 QR 的長度與 MN 的長度相等。

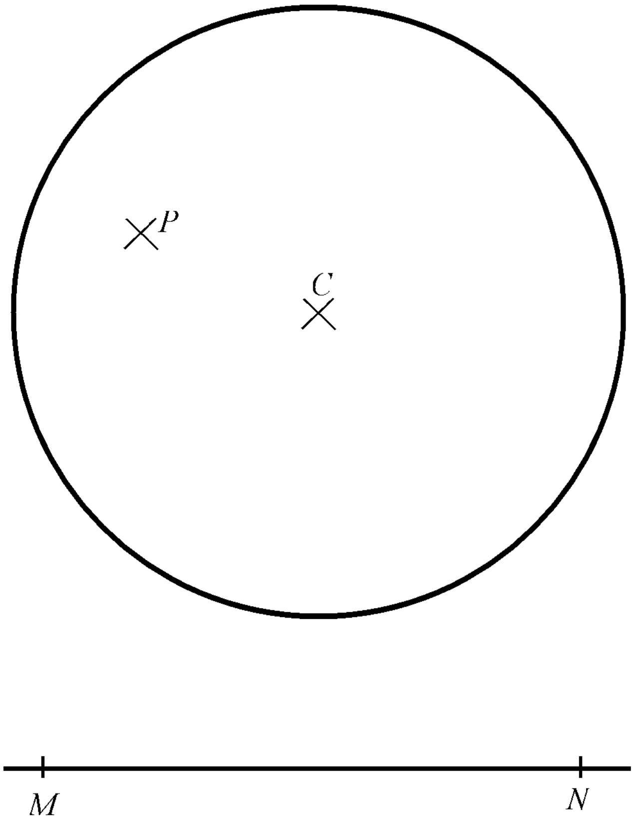
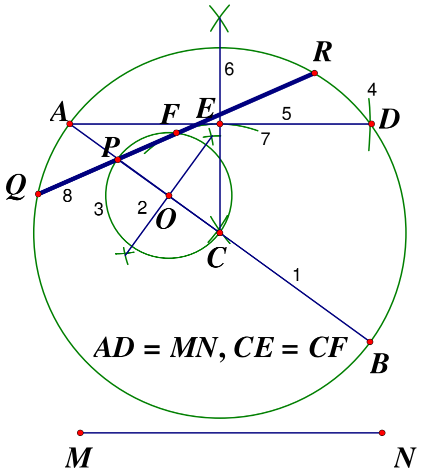


Figure 3
圖三



(1) Join *CP* and produce it to 2 ends *A*, *B* of the diameter of the circle as shown.

(2) Draw the perpendicular bisector of *CP*, *O* is the mid-point of *CP*.

(3) Use *O* as centre, *OC* as radius to draw a circle.

(4) Use *A* as centre, *MN* as radius to draw an arc, cutting the given circle at *D*.

(5) Join *AD*.

(6) Draw the perpendicular bisector of *AD*, *E* is the mid-point of *AD*.

(7) Use *C* as centre, *CE* as radius to draw an arc, cutting the circle in step (3) at *F*.

(8) Join *PF* and produce it to cut the circle at *Q* and *R*. Then *QR* is the required chord.

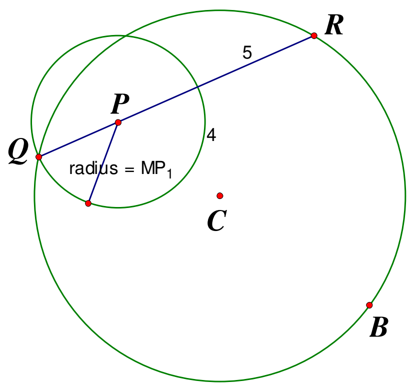
Proof: $\angle PFC = 90°$ ($\angle$ in semi-circle)

$CE = CF$ (by construction)

$QR = AD = MN$ (chords eq. distance from centre are equal)

Method 2: (Provided by Tsuen Wan Government Secondary School Tam Lok Him)

Let the radius of the given circle be *R*, the distance between *CP* be *r*.



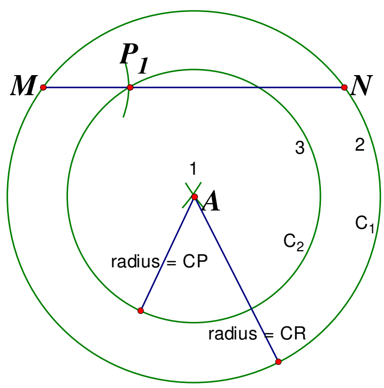
(1) Use *M* as centre and *R* as radius to draw an arc, use *N* as centre and *R* as radius to draw an arc. The two arcs intersect at *A*.

(2) Use *A* as centre, *R* as radius to draw a circle C_1. The circle C_1 must pass through *M*, *N*.

(3) Use *A* as centre, *r* as radius to draw a circle C_2, which cuts *MN* at P_1.

(4) On the given circle, use *P* as centre, MP_1 as radius to draw a circle, which cuts the given circle at *Q*.

(5) Join *QP* and produce it to cut the given circle at *R*.



Then *QPR* is the required chord.

Proof: $QP = MP_1$ (by construction)

$CP = AP_1$ (by construction)

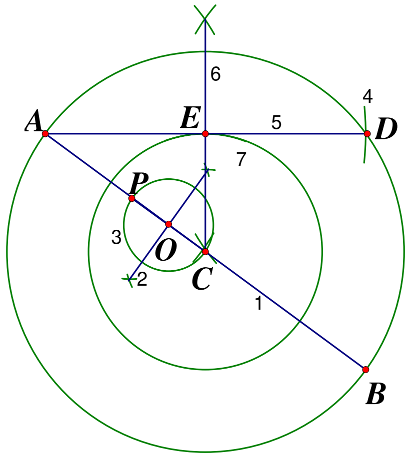
$CQ = AM$ (by construction)

$\Delta CPQ \cong \Delta AP_1M$ (S.S.S.)

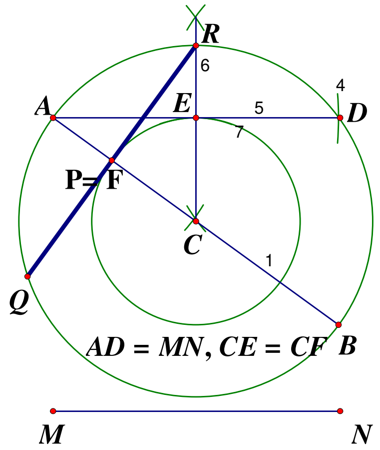
height of ΔCPQ = height of ΔAP_1M

$QR = MN$ (chords eq. distance from centre are equal)

Remark There are some positions of *P* inside the circle for which the chord is not constructible. The circle in step 7 may not cut the circle in step 3. At the limiting position, $CQ^2 = PQ^2 + CP^2$



$$4r^2 = QR^2 + 4CP^2 \Rightarrow QR = 2\sqrt{r^2 - CP^2}$$



$\therefore$ For fixed position *P*, the length of chord must satisfy $2\sqrt{r^2 - CP^2} \le MN \le 2r$.