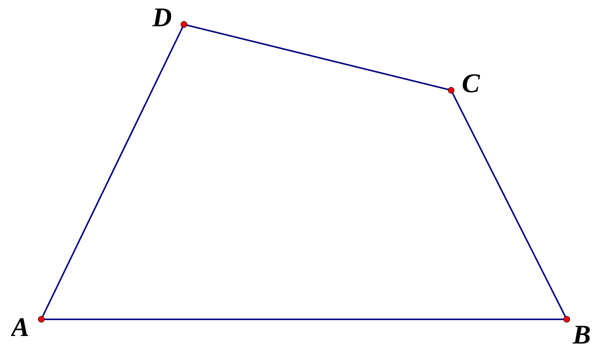
Bisect the area of a quadrilateral

Created by Mr. Francis Hung on 20110119. Last updated: 03 July 2023.

Given a quadrilateral *ABCD*, draw a line segment through a vertex to bisect the area of *ABCD*.



Construction steps

(1) Join the diagonals AC and BD.

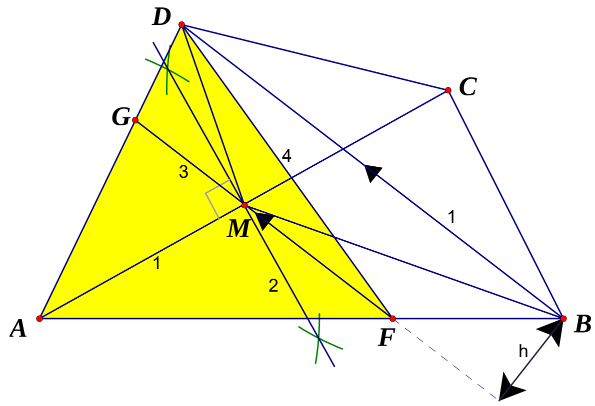
Without loss of generality assume $AC \geq BD$.

(2) Draw the perpendicular bisector of AC. M is the mid-point.

(3) Draw a line segment FG // BD, cutting AB and AD at F and G respectively.

(4) Join DF.

Then $S_{\Delta ADF} = \frac{1}{2} S_{ABCD}$.



Proof: Join BM, DM and let the perpendicular distance between the parallel lines BD and FG be h.

BM = median of ΔABC and DM = median of ΔACD

$S_{\Delta ABM} = \frac{1}{2} S_{\Delta ABC}$ and $S_{\Delta ADM} = \frac{1}{2} S_{\Delta ACD}$

$S_{\Delta ABM} + S_{\Delta ADM} = \frac{1}{2} S_{\Delta ABCD} \cdots\cdots (1)$

Let $GM = x$, $FM = y$

$S_{\Delta BFM} + S_{\Delta DGM} = \frac{1}{2} xh + \frac{1}{2} yh = \frac{1}{2}(x+y)h = \frac{1}{2} FG \cdot h = S_{\Delta DFG}$

$S_{\Delta ADF} = S_{\Delta AFG} + S_{\Delta DFG} = S_{\Delta AFG} + S_{\Delta BFM} + S_{\Delta DGM}$

$= S_{\Delta ABM} + S_{\Delta ADM} = \frac{1}{2} S_{\Delta ABCD}$ by (1)

The proof is completed.