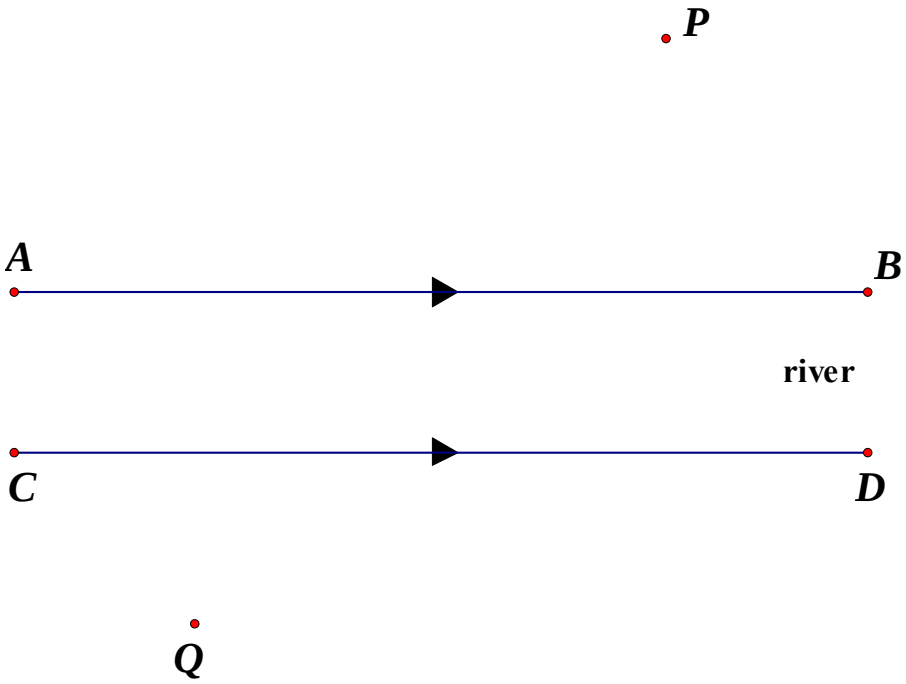
# Bridge

Created by Mr. Francis Hung on 20111008. Last updated: 2023-07-03

In the following diagram, towns $P$ and $Q$ are situated in the opposite shores of two parallel river banks $AB$ and $CD$. You are asked to build a bridge across the river, which must be perpendicular to the river. In order to minimise the walking distance from $P$ to $Q$ via the bridge, what is the best position to build the bridge? Show your works clearly.



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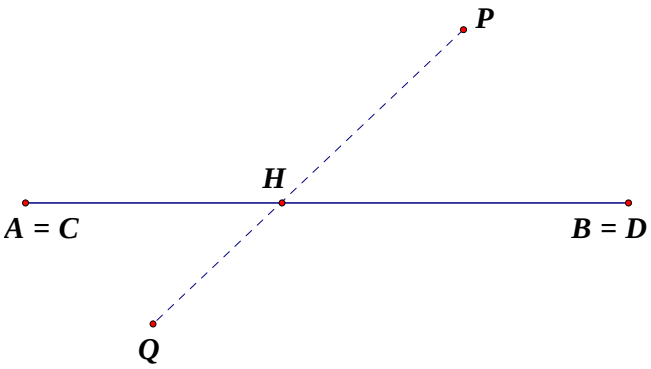
Consider the special case:

When the width of the river is zero, then the diagram is as follows:




In order to minimise the walking distance from $P$ to $Q$ via the bridge, we join $P$ and $Q$ to meet $AB$ (hence $CD$) at $H$. Then $P$, $H$, $Q$ are collinear. $PH + HQ$ will be the minimum. $H$ is the location of the bridge.

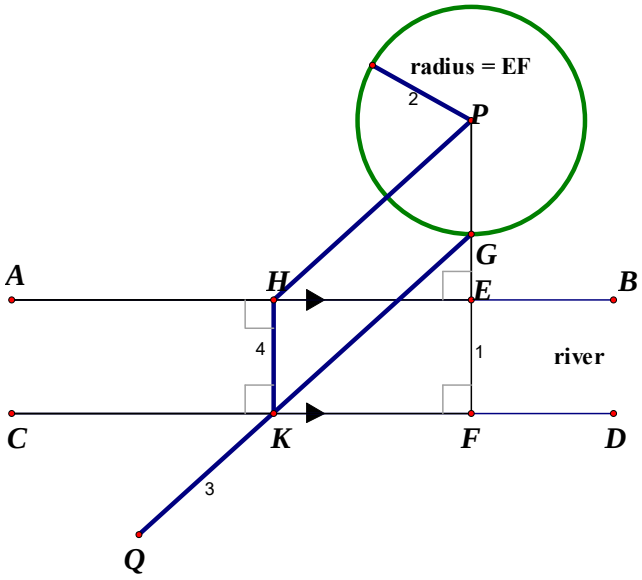
The solution to the general case is as follows:

(1) From $P$ draw a line $PEF \perp AB$. $E$ and $F$ are the feet of perpendiculars on $AB$ and $CD$ respectively.

(2) Use $P$ as centre, $EF$ as radius to draw a circle, cutting $PEF$ at $G$.

(3) Join $QG$, which cuts $CD$ at $K$.

(4) Draw $HK \perp AB$, where $H$ lies on $AB$.




Then $HK$ is the position of the bridge.

Proof: Join $PH$.

$HK = EF = PG$ by construction

$HK \parallel PG$ by construction

$\therefore$ $HKGP$ is a parallelogram

(opp. sides are eq. and parallel)

$PH = GK$ (opp. sides of parallelogram)

$G$, $K$, $Q$ are collinear.

$PH + KQ = GK + KQ$ which is the minimum distance between $G$ and $Q$.

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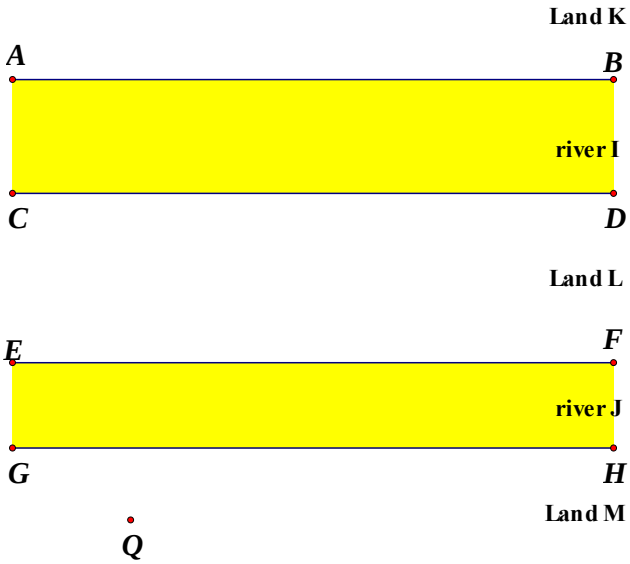
**Follow-up question:**




As shown in the diagram, towns $P$, $Q$ are separated by two parallel rivers $I$ and $J$. The widths of river are $i$, $j$ respectively. You are asked to build two bridges across the rivers. Each one must be perpendicular to the rivers. In order to minimise the walking distance from $P$ to $Q$ via the bridges, what is the best position to build the two bridges? Show your works clearly.




Solution

(1) From $P$ draw a line $PW \perp GH$. $T$, $U$, $V$ and $W$ are the feet of perpendiculars on $AB$, $CD$, $EF$ and $GH$ respectively.

(2) Use $P$ as centre, $TU + VW$ as radius to draw a circle, cutting $PW$ at $M$.

(3) Join $QG$, which cuts $GH$ at $N$.

(4) Draw $NO \perp EF$, where $O$ lies on $EF$.

(5) $OR$ // $QM$, cutting $CD$ at $R$.

(6) Draw $RS \perp AB$, where $S$ lies on $AB$.

(7) Join $PS$.

$SR$ and $ON$ are the positions of the bridges.

Proof: omitted.