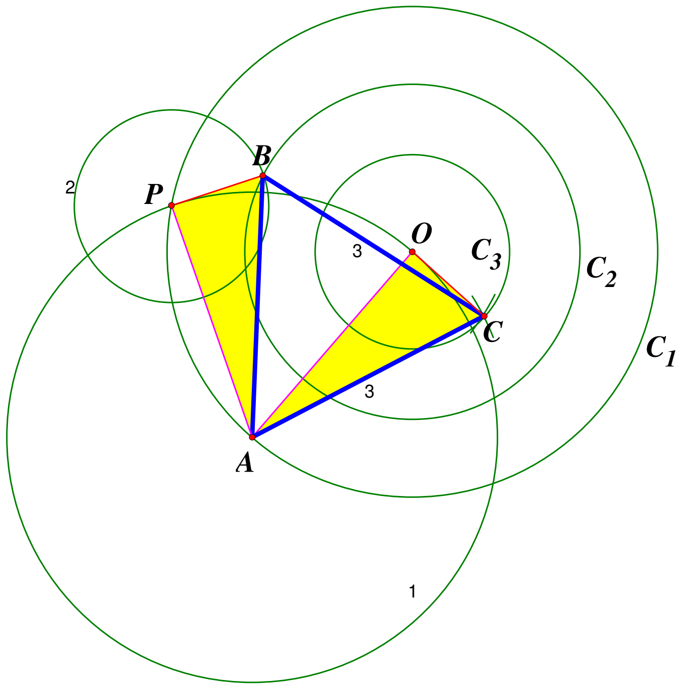
To construct an equilateral triangle on the 3 concentric circles C_1, C_2, C_3 whose radii are a, b, c, where $a > b > c$ and $b + c \geq a$ with a common centre O.

Created by Mr. Francis Hung on 20110119. Last updated: 2021-09-29



Let A be any point on the largest circle.

(1) Use A as centre, a as radius to draw a circle cutting C_1 at P.

(2) Use P as centre, c as radius to draw a circle cutting C_2 at B.

(3) Rotate B anticlockwise 60° about A to C (O, C lie on the same side of AB)

Join AC, BC. Then ΔABC is the required equilateral triangle.

Proof: By step 3, ΔABC is an equilateral triangle.

It is sufficient to prove that C lies on C_3.

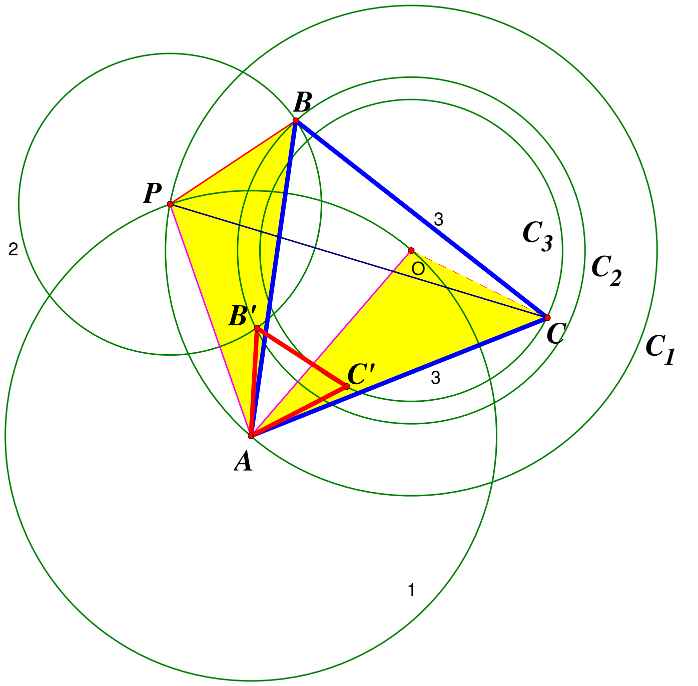
$AB = AC$	(By step 3)
$\angle BAC = 60°$	(By step 3)
$OA = OP$	(radii of C_1)
$AO = AP$	(radii in step 1)
$\therefore$ ΔOAP is an equilateral triangle.	($OA = OP = AP$)
$\angle OAP = 60° = \angle BAC$	
Let $\angle BAP = x$	
$\angle BAO = 60° - x$	
$\therefore$ $\angle CAO = x$	
$\Delta ABP \cong \Delta ACO$	(S.A.S.)
$PB = CO$	(corr. sides, $\cong$ Δ's)

C lies on the circle C_3.

The proof is completed.

Remark

(1) The circles in step 1 and step 2 cut C_1 and C_2 at two different points. There are more than one possible equilateral triangles ABC with different sizes.



(2) If $b + c < a$, then ΔOBP cannot be formed, we cannot construct the equilateral triangle.

(3) At limiting position, $b + c = a$, the circle in step 2 touches C_2 at B. O, B, P are collinear.

The above proof remains vaild.