1991 FG6.1-2

某兩位數 x 之個位數字是 M，十位數字是 N。另一兩位數 y 之個位數字是 N，十位數字是 M。若 $x > y$，且他們的和是他們的差的十一倍，求 M 及 N 的值。

A 2-digit number x has M as the units digit and N as the tens digit. Another 2-digit number y has N as the units digit and M as the tens digit. If $x > y$ and their sum is equal to eleven times their differences, find the values of M and N.

1993 FG8.1-2

已知方程 $x^2 + (m+1)x - 2 = 0$ 有兩整數根$(\alpha + 1)$及$(\beta + 1)$，且$\alpha < \beta$及 $m \neq 0$。設 $d = \beta - \alpha$。求 m 及 d 的值。

Given that the equation $x^2 + (m + 1)x - 2 = 0$ has 2 integral roots $(\alpha + 1)$ and $(\beta + 1)$ with $\alpha < \beta$ and $m \neq 0$. Let $d = \beta - \alpha$. Find the values of m and d.

1995 FI3.3

已知方程 $x + 6 + 8k = k(x + 8)$有正整數解。求 k 的最小值 c。

It is given that the equation $x + 6 + 8k = k(x + 8)$ has positive integral solution. Find c, the least value of k.

1996 HG8

若質數 a、b 為二次方程 $x^2 - 21x + t = 0$ 的根，求 $\left(\frac{b}{a} + \frac{a}{b}\right)$ 的值。

If prime numbers a, b are the roots of the quadratic equation $x^2 - 21x + t = 0$, find the value of $\left(\frac{b}{a} + \frac{a}{b}\right)$.

1996 FG7.1

若方程 $ax^2 - mx + 1996 = 0$ 的兩個不等根是質數，求 a 的值。

If the two distinct roots of the equation $ax^2 - mx + 1996 = 0$ are primes, find the value of a.

1999 FG2.4

設 d 為奇質數，若 $89 - (d + 3)^2$ 是一整數之平方，求 d 之值。

Let d be an odd prime number.
If $89 - (d + 3)^2$ is the square of an integer, find the value of d.

2001 HI6

88 張成人車票總值為 \$□293□，由於列印機壞了，五位數字的首尾兩個數字印不出來。已知每張車票的價值為 \$$P$，其中 P 為一整數，求 P 的值。

The total cost for 88 tickets was \$□293□. Because the printing machine was not functioning well, the first and the last digits of the 5-digit number were missing. If the cost for each ticket is \$$P$, where P is an integer, find the value of P.

2001 FG3.2

已知方程 $x^2y - x^2 - 3y - 14 = 0$ 只得一組正整數解 (x_0, y_0)。若 $x_0 + y_0 = b$，求 b 的值。

Suppose the equation $x^2y - x^2 - 3y - 14 = 0$ has only one positive integral solution (x_0, y_0). If $x_0 + y_0 = b$, find the value of b.

2001 FG4.4

方程 $x^2 - 45x + m = 0$ 的兩個根皆為質數。
已知兩根的平方和為 d，求 d 的值。

The roots of the equation $x^2 - 45x + m = 0$ are prime numbers.
Given that the sum of the squares of the roots is d, find the value of d.

2004 FG3.2

已知質數 p 和 q 滿足方程 $18p + 30q = 186$。
若 $\log_8 \frac{p}{3q+1} = b \geq 0$，求 b 的值。

Given that p and q are prime numbers satisfying the equation $18p + 30q = 186$.
If $\log_8 \frac{p}{3q+1} = b \geq 0$, find the value of b.

2005 FG1.2

設質數 p 和 q 是方程 $x^2 - 13x + R = 0$ 的兩個不同的根，其中 R 是實數。若 $b = p^2 + q^2$，求 b 的值。

Let p and q be prime numbers that are the two distinct roots of the equation $x^2 - 13x + R = 0$, where R is a real number. If $b = p^2 + q^2$, find the value of b.

2008 HI3

已知 x_0 及 y_0 為正整數且滿足方程 $\frac{1}{x} + \frac{1}{y} = \frac{1}{15}$。
若 $35 < y_0 < 50$ 及 $x_0 + y_0 = z_0$，求 z_0 的值。

Given that x_0 and y_0 are positive integers satisfying the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{15}$.
If $35 < y_0 < 50$ and $x_0 + y_0 = z_0$, find the value of z_0.

2009 HG3

已知 p 和 q 為整數。若 $\frac{2}{p} + \frac{1}{q} = 1$，求 $p \times q$ 的最大值。

Given that p and q are integers. If $\frac{2}{p} + \frac{1}{q} = 1$, find the maximum value of $p \times q$.

2010 HI4

已知 $x+y+z=3$ 及 $x^3+y^3+z^3=3$，且 x, y, z 為整數。

若 $x<0$，求 y 的值。

Given that $x+y+z=3$ and $x^3+y^3+z^3=3$, where x, y, z are integers.
If $x<0$, find the value of y.

2010 HI5

已知 a, b, c, d 為正整數，且滿足 $\log_a b=\frac{1}{2}$ 及 $\log_c d=\frac{3}{4}$。

若 $a-c=9$，求 $b-d$ 的值。

Given that a, b, c, d are positive integers satisfying $\log_a b=\frac{1}{2}$ and $\log_c d=\frac{3}{4}$.

If $a-c=9$, find the value of $b-d$.

2010 FG4.1

設 a 為整數及 $a\neq 1$。已知方程$(a-1)x^2-mx+a=0$ 的兩根均為正整數。

求 m 的值。

Let a be an integer and $a\neq 1$. Given that the equation $(a-1)x^2-mx+a=0$ has two roots which are positive integers. Find the value of m.

2010 FG4.3

已知 A、B、C 為正整數，且 A、B 和 C 的最大公因數等於 1。

若 A、B、C 滿足$A\log_{500}5+B\log_{500}2=C$，求 $A+B+C$ 的值。

Given that A, B, C are positive integers with their greatest common divisor equal to 1. If A, B, C satisfy $A\log_{500}5+B\log_{500}2=C$, find the value of $A+B+C$.

2010 FGS.4

共有多少個正整數 m 使得通過點 $A(-m, 0)$及點 $B(0, 2)$的直綫亦通過 $P(7, k)$，其中 k 為一正整數？

How many positive integers m are there for which the straight line passing through points $A(-m, 0)$ and $B(0, 2)$ and also passes through the point $P(7, k)$, where k is a positive integer?

2011 FI2.1

若方程組$\begin{cases}x+y=P\\3x+5y=13\end{cases}$的解為正整數，求 P 的值。

If the solution of the system of equations $\begin{cases}x+y=P\\3x+5y=13\end{cases}$ are positive integers, find the value of P.

2011 FI2.3

若 a 及 b 為相異質數且 $a^2-5a+R=0$ 及 $b^2-5b+R=0$，求 R 的值。

If a and b are distinct prime numbers and $a^2-5a+R=0$ and $b^2-5b+R=0$, find the value of R.

2011 FGS.4

設 F 為方程 $x^2+y^2+z^2+w^2=3(x+y+z+w)$的整數解的數目。求 F 的值。

Let F be the number of integral solutions of $x^2+y^2+z^2+w^2=3(x+y+z+w)$. Find the value of F.

2012 HI6

已知 a 及 b 為不同質數，且 $a^2-19a+m=0$ 及 $b^2-19b+m=0$，

求 $\frac{a}{b}+\frac{b}{a}$ 的值。

Given that a and b are distinct prime numbers, $a^2-19a+m=0$ and $b^2-19b+m=0$. Find the value of $\frac{a}{b}+\frac{b}{a}$.

2012 HI8

若方程 $(k^2-4)x^2-(14k+4)x+48=0$ 有兩個相異的正整數根，求 k 的值。

If the quadratic equation $(k^2-4)x^2-(14k+4)x+48=0$ has two distinct positive integral roots, find the value(s) of k.

2012 HI9

已知 x、y 為正整數，且 $x>y$，解 $x^3=2189+y^3$。

Given that x, y are positive integers and $x>y$, solve $x^3=2189+y^3$.

2012 HG1

已知 x、y 及 z 為三個連續正整數，且 $\frac{y}{x}+\frac{z}{x}+\frac{x}{y}+\frac{z}{y}+\frac{x}{z}+\frac{y}{z}$ 為整數，

求 $x+y+z$ 的值。

Given that x, y and z are three consecutive positive integers, and $\frac{y}{x}+\frac{z}{x}+\frac{x}{y}+\frac{z}{y}+\frac{x}{z}+\frac{y}{z}$ is an integer. Find the value of $x+y+z$.

2013 HI4

已知 $x^2+399=2^y$，其中 x、y 為正整數。求 x 的值。

Given that $x^2+399=2^y$, where x, y are positive integers. Find the value of x.

2015 FG3.2 2023 FG4.2

設 p 為質數及 m 為整數。若 $p(p+m)+2p=(m+2)^3$，找出 m 的最大可能值。

Let p be a prime and m be an integer.
If $p(p+m)+2p=(m+2)^3$, find the greatest possible value of m.

2016 HI4

若 x、y 為整數，有多少對 x、y 且滿足 $(x+1)^2+(y-2)^2=50$？

If x, y are integers, how many pairs of x, y are there which satisfy the equation

$$(x+1)^2+(y-2)^2=50\ ?$$

2016 HG9

設整數 a、b 及 c 為三角形的邊長。已知 $f(x)=x(x-a)(x-b)(x-c)$，且 x 為一個大於 a、b 及 c 的整數。若 $x=(x-a)+(x-b)+(x-c)$及 $f(x)=900$，求該三角形三條垂高的總和。

Let the three sides of a triangle are of lengths a, b and c where all of them are integers. Given that $f(x)=x(x-a)(x-b)(x-c)$ where x is an integer of size greater than a, b and c.

If $x=(x-a)+(x-b)+(x-c)$ and $f(x)=900$,

find the sum of the lengths of the three altitudes of this triangle .

2017 HG10

已知方程 $a^2x^2-(4a-3a^2)x+2a^2-a-21=0$ (其中 $a>0$) 最少有一個整數根，求所有 a 的可能整數值之和。

It is given that the equation $a^2x^2-(4a-3a^2)x+2a^2-a-21=0$ (where $a>0$) has at least one integral root. Find the sum of all possible integral values of a .

2018 HG6

已知 $n^4+104=3^m$，其中 n、m 為正整數。求 n 的最小值。

Given that $n^4+104=3^m$, where n, m are positive integers.

Find the least value of n .

2023 FI4.2

如果方程組 $\begin{cases} x^2-3xy+2y^2-z^2=31 \\ -x^2+6yz+2z^2=44 \\ x^2+xy+8z^2=100 \end{cases}$ 整數解的數量是 γ，求 γ 的值。

If the system of equations $\begin{cases} x^2-3xy+2y^2-z^2=31 \\ -x^2+6yz+2z^2=44 \\ x^2+xy+8z^2=100 \end{cases}$ has γ sets of integral solutions, find the value of γ .

Answers

1991 FG6.1-2 $M = 4$, $N = 5$	1993 FG8.1-2 $m = -2$, $d = 3$	1995 FI3.3 2	1996 HG8 $\frac{365}{38}$	1996 FG7.1 2
1999 FG2.4 5	2001 HI6 147	2001 FG3.2 20	2001 FG4.4 1853	2004 FG3.2 0
2005 FG1.2 125	2008 HI3 64	2009 HG3 9	2010 HI4 4	2010 HI5 –3
2010 FG4.1 3	2010 FG4.3 6	2010 FGS.4 4	2011 FI2.1 3	2011 FI2.3 6
2011 FGS.4 208	2012 HI6 $\frac{293}{34}$	2012 HI8 4	2012 HI9 $x = 13$, $y = 2$	2012 HG1 6
2013 HI4 25	2015 FG3.2 2023 FG4.2 0	2016 HI4 12	2016 HG9 $\frac{281}{13}$	2017 HG10 11
2018 HG6 5	2023 FI4.2 0			