Individual Events

I1	***a***	258	**I2**	***a***	17	**I3**	***a***	9	**I4**	***x***	$\frac{3}{2}$	**I5**	***a***	360
	b	15		***b***	136		***b***	15		***y***	1		***b***	36
	c	225		***c***	15		***c***	11		***z***	8		***c***	54
	d	75		**probability**	$\frac{1}{5}$		***d***	73.5		$\log_z y$	0		***d***	36

Group Events

G6	log 6	$a + b$	**G7**	surface area	320π cm^2	**G8**	⊔ L ⊐ ⊓ ⊔	23485	**G9**	***A***	15	**G10**	No. of digits	10
	$3.5a+3.5c$	3.5		volume	$\frac{2000\pi}{3}$ cm^3		ans	⌝ ⊔ ⌜		***B***	56		smaller no.	63
	$\frac{\log 30}{\log 15}$	$\frac{a+b+c}{b+c}$ or $\frac{b+1}{b+1-a}$		volume	$\frac{2}{3}\pi r^2 h$		word	CHRISTMAS		***C***	8		bigger no.	65
	$(\log 15)^2 - \log 15$	$(b+c)(b+c-1)$ or $(b-a+1)(b-a)$		ratio	3 : 1		message	JOIN US		***X***	0		bigger no.	$3^{\frac{1}{3}} \times 8^{\frac{1}{8}}$

Individual Event 1

I1.1 Find a if $a = 5 + 8 + 11 + \ldots + 38$.
This is an arithmetic series with first term = 5, common difference = 3
Let n be the number of terms. $38 = 5 + (n - 1)(3) \Rightarrow n = 12$
$a = \frac{1}{2}(5 + 38)\cdot 12 = 258$

I1.2 Let b = the sum of the digits of the number a. Find b.
$b = 2 + 5 + 8 = 15$

I1.3 If $c = b^2$, find c.
$c = 15^2 = 225$

I1.4 Given that $3d = c$, find d.
$3d = 225 \Rightarrow d = 75$
Put $x = -4$ into the polynomial: $2(-4)^2 + 3(-4) + 4d = 0$
$d = -5$

Individual Event 2

I2.1 Two cards are drawn at random from a pack and not replaced.
If the probability that both cards are hearts is $\frac{1}{a}$, find a.
$\text{P(both hearts)} = \frac{1}{a} = \frac{13}{52} \times \frac{12}{51} = \frac{1}{17}$
$a = 17$

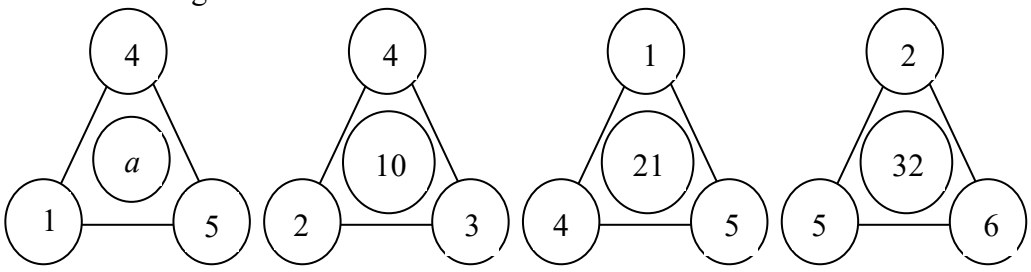
I2.2 If there are b ways of choosing 15 people from 'a' people, find b.
$b = C_{15}^{17} = \frac{17 \times 16}{2} = 136$

I2.3 If c signals can be made with $\frac{b}{2a}$ flags of different colours by raising at least one of the flags, without considering the arrangement of colours, find c.
$\frac{b}{2a} = \frac{136}{2 \cdot 17} = 4$
The following are different patterns:
0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111,
where '0' in the ith position represents the ith colour flag is put down and '1' represents the ith colour flag is raised.
$c = 15$

I2.4 There are c balls in a bag, of which 3 are red. What is the probability of drawing a red ball?
$\text{P(red ball)} = \frac{3}{15} = \frac{1}{5}$

Individual Event 3

I3.1 Find a in the figure.



$(1 \times 5) + 4 = 9$ $(2 \times 3) + 4 = 10$ $(4 \times 5) + 1 = 21$ $(5 \times 6) + 2 = 32$.

$a = 9$

I3.2 Find b if $\dfrac{\sin(4b)^\circ}{\cos(4b)^\circ} = \sqrt{\sqrt{a}}$ $(0 < 4b < 90)$

$\tan (4b)^\circ = \sqrt{3}$

$4b = 60 \Rightarrow b = 15$

I3.3 Find c from the sequence: $\dfrac{3}{12}, \dfrac{7}{34}, \dfrac{c}{56}, \dfrac{b}{78}$.

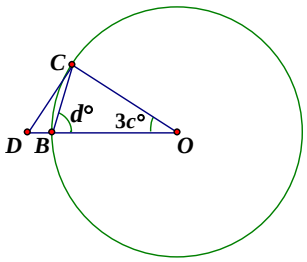
$\dfrac{3}{12}, \dfrac{7}{34}, \dfrac{c}{56}, \dfrac{15}{78}$

$12 + 22 = 34$, $34 + 22 = 56$, $56 + 22 = 78$

$3 + 4 = 7$, $7 + 4 = 11$, $11 + 4 = 15$

$c = 11$

I3.4 In the figure, O is the centre, B and C are points on the circumference. $\angle BOC = 3c^\circ$, $\angle OBC = d^\circ$. Find d.



$\angle BCO = d^\circ$ (base $\angle$s isos. Δ)

$2d + 33 = 180$ ($\angle$s sum of Δ)

$d = 73.5$

Individual Event 4

I4.1 Find x if $x = \dfrac{\log a^3}{\log a^2}$ where $a > 0$ and $a \neq 1$.

$$x = \frac{\log a^3}{\log a^2} = \frac{3\log a}{2\log a} = \frac{3}{2}$$

I4.2 If $y - 1 = \log x + \log 2 - \log 3$, find y.

$y - 1 = \log \dfrac{3}{2} + \log 2 - \log 3$

$y = \log\left(\dfrac{3}{2} \times \dfrac{2}{3}\right) + 1 = \log 1 + 1 = 1$

I4.3 What is Z if $\log_2 Z^y = 3$?

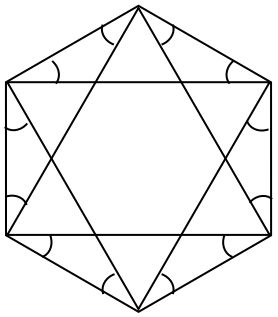
$\log_2 Z = 3 \Rightarrow Z = 2^3 = 8$

I4.4 Find $\log_z y$.

$\log_8 1 = 0$

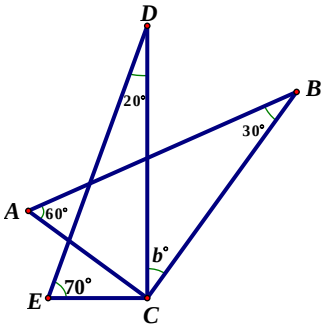
Individual Event 5

I5.1 Let the sum of the marked angles be $a°$. Find a.



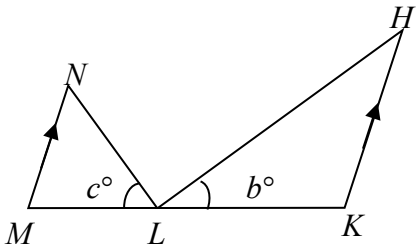
The figure shows two equilateral triangles inscribed in a regular hexagon.
Each interior angle of the hexagon = 120°
Each angle of an equilateral triangle = 60°
Each marked angle = (120° – 60°)÷ 2 = 30°
There are 12 marked angles.
$a = 12\times30 = 360$

I5.2 $\angle ACE = \left(\frac{a}{10}\right)^\circ$. Find b.



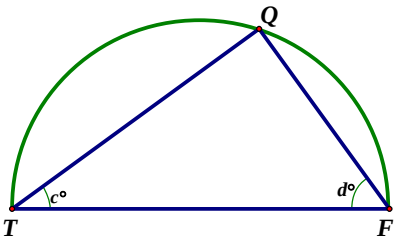
$\angle DCE = 180° – 20° – 70° = 90°$ (∠s sum of Δ)
$\angle ACE = 36°$
$\angle ACD = 90° – 36° = 54°$
$\angle ACB = 180° – 30° – 60° = 90°$ (∠s sum of Δ)
$b = 90 – 54 = 36$

I5.3 If $HK = KL$, $LM = MN$, $HK \parallel MN$, find c.



$\angle KHL = b° = 36°$ (base ∠s isos. Δ)
$\angle LKH = 180° – 36° – 36° = 108°$ (∠s sum of Δ)
$\angle LMN = 180° – 108° = 72°$ (int. ∠s, $NM \parallel HK$)
$\angle MNL = c°$ (base ∠s isos. Δ)
$c + c + 72 = 180$ (∠s sum of Δ)
$c = 54$

I5.4 TQF is a semi-circle. Find d.



$\angle TQF = 90°$ (∠s in semi-circle)
$c + d = 90$ (∠s sum of Δ)
$54 + d = 90$
$d = 36$

Group Event 6

Let $\log 2 = a$, $\log 3 = b$, $\log 5 = c$.

G6.1 Express $\log 6$ in terms of a, b and c.
$\log 6 = \log 2 + \log 3 = a + b$

G6.2 Evaluate $3.5\,a + 3.5\,c$.
$3.5\,a + 3.5\,c = 3.5\log 2 + 3.5\log 5$
$= 3.5\log(2\times5) = 3.5$

G6.3 Express $\frac{\log30}{\log15}$ in terms of a, b and c.

$$\frac{\log30}{\log15} = \frac{\log3+\log10}{\log3+\log10-\log2} = \frac{b+1}{b+1-a} \quad \text{or} \quad \frac{\log30}{\log15} = \frac{\log2+\log3+\log5}{\log3+\log5} = \frac{a+b+c}{b+c}$$

G6.4 Express $(\log 15)^2 – \log 15$ in terms of a, b and c.
$(\log 15)^2 – \log 15 = \log 15(\log 15 – 1) = (\log 3 + \log 10 – \log 2)(\log 3 – \log 2)$
$= (b – a + 1)(b – a)$
OR $(\log 15)^2 – \log 15 = \log 15(\log 15 – 1) = (\log 3 + \log 5)(\log 3 + \log 5 – 1) = (b+c)(b+c–1)$

Group Event 7

G7.1 Figure 1 shows a cone and a hemisphere.
OB = 12 cm, r = 10 cm. Express the surface area of the solid in terms of π.
The surface area = $2\pi r^2 + \pi rL = 320\pi$ cm^2

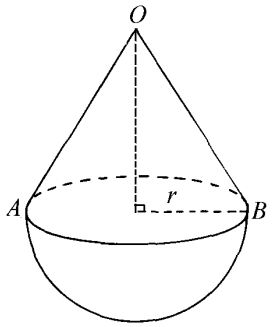


Figure 1

G7.2 What is the volume of the hemisphere shown in figure 1? Give your answer in terms of π.
Volume $= \frac{2}{3}\pi r^3 = \frac{2000\pi}{3}$ cm^3

G7.3 In figure 2, a right circular cone stands inside a right cylinder of same base radius r and height h. Express the volume of the space between them in terms of r and h.
Volume of space $= \pi r^2 h - \frac{1}{3}\pi r^2 h$
$= \frac{2}{3}\pi r^2 h$

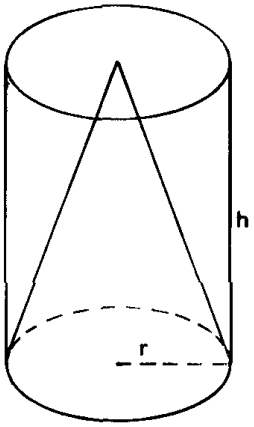


Figure 2

G7.4 Find the ratio of the volume of the cylinder to that of the cone.
Ratio $= \pi r^2 h : \frac{1}{3}\pi r^2 h = 3 : 1$

Group Event 8

Given that:

1	2	3
4	5	6
7	8	9

1 stands for *A*
2 stands for *B*
……
25 stands for *Y*
26 stands for *Z*

G8.1 What number does the code ㄩ ㄴ ㄱ ㄇ ㅁ stand for?
ㄩ ㄴ ㄱ ㄇ ㅁ stands for 23485

G8.2 Put Δ stands for zero. Calculate the following and give the answer in code.
(ㄩΔ)(ㄱ Δ) + ㄹ ㄲ – ㄴΔ
= 20×40 + 19 – 30 = 789
= ㄱ ㅁ ㄲ

G8.3 "3 8 18 9 19 20 13 1 19" stands for a word. What is it?
3 = C, 8 = H, 18 = R, 9 = I, 19 = S, 20 = T, 13 = M, 1 = A, 19 = S
The number stands for "CHRISTMAS"

G8.4 Decode the following message: (ㄹ Δ ㄹ ㅁ ㄲ ㄹ ㄱ) (ㄩ ㄹ ㄹ ㄲ)
There are two words in the message.
(ㄹ Δ ㄹ ㅁ ㄲ ㄹ ㄱ) (ㄩ ㄹ ㄹ ㄲ) = (10 15 9 14) (21 19) = JOIN US

Group Event 9

G9.1 Find A from the sequence: 0, 3, 8, A, 24, 35, …

$1^2 - 1, 2^2 - 1, 3^2 - 1, 4^2 - 1, 5^2 - 1, 6^2 - 1, \ldots$ $A = 4^2 - 1 = 15$

G9.2 The roots of the equation $x^2 - Ax + B = 0$ are 7 and C. Find B and C.

$x^2 - 15x + B = 0$

$7 + C = 15 \Rightarrow C = 8$

$B = 7C = 56$

G9.3 If $\log_7 B = \log_7 C + 7^X$; find X.

$\log_7 56 = \log_7 8 + 7^X$

$7^X = \log_7 (56/8) = \log_7 7 = 1$

$X = 0$

Group Event 10

G10.1 How many digits are there in the number N if $N = 2^{12} \times 5^8$?

Reference: 1992HI17, 2012 HI4

$N = 2^{12} \times 5^8 = 2^4 \times 10^8 = 16 \times 10^8$

There are 10 digits.

G10.2 If $(2^{48} - 1)$ is divisible by two whole numbers between 60 and 70, find them.

$2^{48} - 1 = (2^{24} + 1)(2^{24} - 1) = (2^{24} + 1)(2^{12} + 1)(2^{12} - 1) = (2^{24} + 1)(2^{12} + 1)(2^6 + 1)(2^6 - 1)$

Smaller number = $2^6 - 1 = 63$, larger number = $2^6 + 1 = 65$.

G10.3 Given $2^{\frac{1}{2}} \times 9^{\frac{1}{9}}$, $3^{\frac{1}{3}} \times 8^{\frac{1}{8}}$. What is the greatest number?

$$2^{\frac{1}{2}} \times 9^{\frac{1}{9}} = 2^{\frac{1}{2}} \times 3^{\frac{2}{9}}; \quad 3^{\frac{1}{3}} \times 8^{\frac{1}{8}} = 3^{\frac{1}{3}} \times 2^{\frac{3}{8}}$$

$$\frac{2^{\frac{1}{2}} \times 3^{\frac{2}{9}}}{3^{\frac{1}{3}} \times 2^{\frac{3}{8}}} = \frac{2^{\frac{1}{8}}}{3^{\frac{1}{9}}} = \frac{\left(2^9\right)^{\frac{1}{72}}}{\left(3^8\right)^{\frac{1}{72}}} = \left(\frac{512}{6561}\right)^{\frac{1}{72}} < 1$$

$\therefore$ $3^{\frac{1}{3}} \times 8^{\frac{1}{8}}$ is the greatest.