Individual Events

SI	*a*	2	I1	*a*	5	I2	*a*	125	I3	*a*	4	I4	*a*	2	I5	*t*	8
	b	54		*b*	0		*b*	15		*b*	16		*b*	10		*u*	135
	c	2		*c*	–9		*c*	3		*c*	199		*c*	96		*v*	45
	d	1		*d*	2		*d*	16		*d*	4		*d*	95		*w*	70

Group Events

SG	*s*	19	G6	*x*	8	G7	*M*	100	G8	*M*	5	G9	*A*	60	G10	*k*	15
	n	8		*y*	25		*N*	59		*N*	2		*r*	3		*C*	6
	K	$\frac{1}{50}$		*d*	4		*x*	$\frac{24}{5}$		*x*	170		*n*	20		*R*	8
	A	200		*h*	$\frac{12}{5}$		*S*	1		*y*	5000		*x*	3240		*A*	243

Sample Individual Event (1994 Final Sample Individual Event)

SI.1 The sum of two numbers is 40, and their product is 20.

If the sum of their reciprocals is a, find a.

Reference: 1983 FG6.3, 1984 FSG.1, 1986 FSG.1

Let the two numbers be x and y.

$x + y = 40$ and $xy = 20$

$$a = \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = 2$$

SI.2 If b cm^2 is the total surface area of a cube of side $(a + 1)$ cm, find b.

Similar Questions: 1984 FI3.2, 1984 FG9.2

$a + 1 = 3$

$b = 6\times 3^2 = 54$

SI.3 One ball is taken at random from a bag containing $b - 4$ white balls and $b + 46$ red balls.

If $\frac{c}{6}$ is the probability that the ball is white, find c.

There are $b - 4 = 50$ white balls and $b + 46 = 100$ red balls

$$\text{P(white ball)} = \frac{50}{50+100} = \frac{2}{6} \Rightarrow c = 2$$

SI.4 The length of a side of an equilateral triangle is c cm. If its area is $d\sqrt{3}$ cm^2, find d.

Reference: 1984FI4.4, 1986 FSG.3, 1987 FG6.2, 1988 FG9.1

$$d\sqrt{3} = \frac{1}{2}\cdot c^2 \sin 60^\circ = \sqrt{3}$$

$d = 1$

Individual Event 1

I1.1 Find a if $a=\log_5\frac{(125)(625)}{25}$.

$a=\log_5\frac{5^3\cdot 5^4}{5^2}=\log_5 5^5$

$a = 5$

I1.2 If $\left(r+\frac{1}{r}\right)^2=a-2$ and $r^3+\frac{1}{r^3}=b$, find b.

Reference: 1990 HI12, 2017 FI1.4

$\left(r+\frac{1}{r}\right)^2=r^2+2+\frac{1}{r^2}=3\Rightarrow r^2+\frac{1}{r^2}=1$

$b=r^3+\frac{1}{r^3}=\left(r+\frac{1}{r}\right)\left(r^2-1+\frac{1}{r^2}\right)=\left(r+\frac{1}{r}\right)(1-1)=0$

I1.3 If one root of the equation $x^3+cx+10=b$ is 2, find c.

Put $x = 2$ into the equation: $8 + 2c + 10 = 0$

$c = -9$

I1.4 Find d if $9^{d+2}=(6489+c)+9^d$. **(Reference: 1986 FG7.4)**

$81\times 9^d=6480+9^d$

$80\times 9^d=6480\Rightarrow 9^d=81$

$d = 2$

Individual Event 2

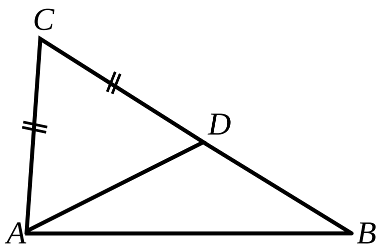
I2.1 Find a in the following sequence: 1, 8, 27, 64, a, 216, ……

$1^3, 2^3, 3^3, 4^3, a, 6^3, \ldots\ldots$

$a=5^3=125$

I2.2 In Figure 1, $AC = CD$ and $\angle CAB-\angle ABC=(a-95)^\circ$.

If $\angle BAD = b^\circ$, find b. **(Reference: 2010 HG3)**



Let $\angle CAD=\theta=\angle CDA$ (base $\angle$s isosceles Δ)

$\angle CAB=b^\circ+\theta$

$\angle CAB-\angle ABC=30^\circ\Rightarrow\angle ABC=b^\circ+\theta-30^\circ$

$\angle BAD+\angle ABC=\angle CDA$ (ext. $\angle$ of Δ)

$b^\circ+b^\circ+\theta-30^\circ=\theta\Rightarrow b=15$

I2.3 A line passes through the points $(-1, 1)$ and $(3, b-6)$. If the y-intercept of the line is c, find c.

Similar question: 1986 FI1.4

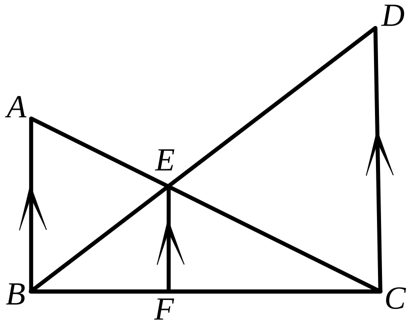
$b-6=9$

$\frac{c-9}{0-3}=\frac{9-1}{3-(-1)}$

$c = 3$

I2.4 In Figure 2, $AB = c + 17$, $BC = 100$, $CD = 80$.

If $EF = d$, find d. **(Reference: 1989 HG8, 1990 FG6.4)**



Let $BF = x$, then $FC = 100 - x$.

$\Delta BEF\sim\Delta BDC$ (equiangular)

$\Delta CEF\sim\Delta CAB$ (equiangular)

$\frac{x}{d}=\frac{100}{80}$ ……(1), $\frac{100-x}{d}=\frac{100}{3+17}$ ……(2)

(1) + (2): $\frac{100}{d}=100\cdot\left(\frac{1}{80}+\frac{1}{20}\right)\Rightarrow d=16$

Individual Event 3

I3.1 The acute angle formed by the hands of a clock at 2:15 is $\left(18\frac{1}{2}+a\right)^{\circ}$. Find a.

Reference: 1984 FG7.1, 1987 FG7.1, 1989 FI1.1, 1990 FG6.3, 2007 HI1

At 2:00, the angle between the arms of the clock = 60°

From 2:00 to 2:15, the hour-hand had moved $360°\times\frac{1}{12}\times\frac{1}{4}$ = 7.5°

The minute hand had moved 90°

$18.5 + a = 90 - (60 + 7.5) = 22.5$

$a = 4$

I3.2 If the sum of the coefficients in the expansion of $(x + y)^a$ is b, find b.

Put $x = 1$ and $y = 1$, then $b = (1 + 1)^4 = 16$

I3.3 If $f(x) = x - 2$, $F(x, y) = y^2 + x$ and $c = F(3, f(b))$, find c.

Reference: 1990 HI3, 2013 FI3.2, 2015 FI4.3

$f(b) = 16 - 2$

$= 14$

$c = F(3, 14)$

$= 14^2 + 3$

$= 199$

I3.4 x, y are real numbers. If $x + y = c - 195$ and d is the maximum value of xy, find d.

Reference: 1988 FI4.3

$x + y = 4$

$\Rightarrow y = 4 - x$

$xy = x(4 - x) = -(x - 2)^2 + 4 \le d$

$\Rightarrow d = 4$

Individual Event 4

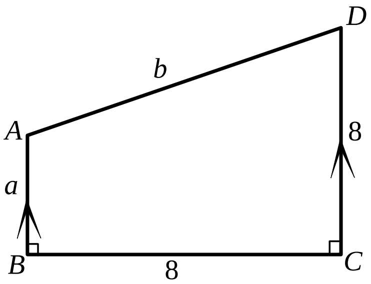
I4.1 If the lines $x + 2y + 3 = 0$ and $4x - ay + 5 = 0$ are perpendicular to each other, find a.

Reference: 1983 FG9.3, 1984 FSG.3, 1986 FSG.2, 1987 FG10.2, 1988 FG8.2

$$-\frac{1}{2}\times\frac{4}{a}=-1$$

$\Rightarrow a = 2$

I4.2 In Figure 1, $ABCD$ is a trapezium with AB parallel to DC and $\angle ABC = \angle DCB = 90°$. If $AB = a$, $BC = CD = 8$ and $AD = b$, find b.



Draw a line segment AE // BC, cutting DC at E.

$\angle BAE = 90° = \angle AEC$ (int. $\angle$s, AE // BC)

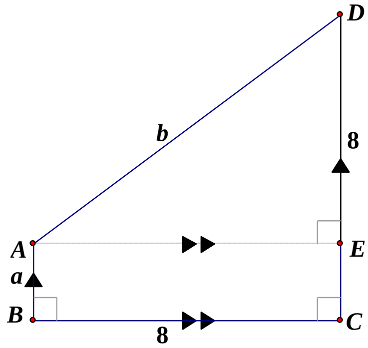
$ABCE$ is a rectangle

$AE = 8$, $CE = a = 2$ (opp. sides, //-gram)

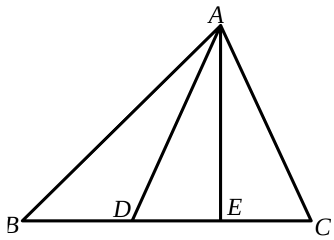
$DE = 8 - a = 6$

$b^2 = 8^2 + 6^2 = 100$ (Pythagoras' theorem on ΔADE)

$b = 10$



I4.3 In Figure 2, $BD = \frac{b}{2}$, $DE = 4$, $EC = 3$. If the area of ΔAEC is 24 and the area of ΔABC is c, find c.



ΔABD, ΔADE and ΔACE have the same height.

The area of $\Delta ABC = c = 24\times\frac{5+4+3}{3} = 96$

I4.4 If $3x^3 - 2x^2 + dx - c$ is divisible by $x - 1$, find d.

$3 - 2 + d - 96 = 0$

$d = 95$

Individual Event 5

I5.1 If $1 + 2 + 3 + 4 + \dots + t = 36$, find t.

$\frac{1}{2}\cdot t(t+1) = 36$

$t = 8$ or -9 (rejected)

I5.2 If $\sin u° = \frac{2}{\sqrt{t}}$ and $90 < u < 180$, find u.

$\sin u° = \frac{1}{\sqrt{2}}$

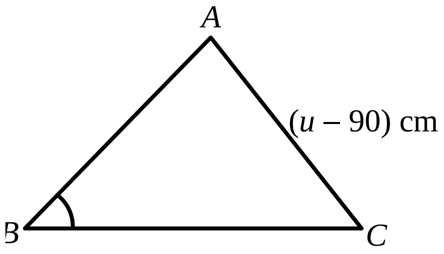
$\Rightarrow u = 135$

I5.3 In Figure 1, $\angle ABC = 30°$ and $AC = (u - 90)$ cm.

If the radius of the circumcircle of ΔABC is v cm, find v.

$\frac{135-90}{\sin 30^\circ} = 2v$ (Sine formula)

$v = 45$



I5.4 In Figure 2, ΔPAB is formed by the 3 tangents of the circle with centre O. If $\angle APB = (v - 5)°$ and $\angle AOB = w°$, find w.

$\angle APB = 40°$

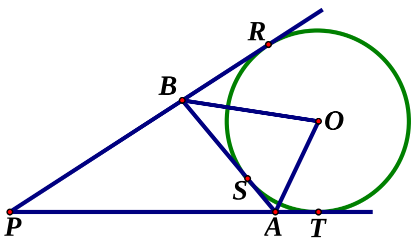
$OT \perp PA$, $OS \perp AB$, $OR \perp PB$ (tangent $\perp$ radius)

$\angle ROT = 360° - 40° - 90° - 90° = 140°$ ($\angle$s sum of polygon)

$\angle ROB = \angle SOB$, $\angle TOA = \angle SOA$ (tangent from ext. pt.)

$\angle AOB = 140° \div 2 = 70°$

$\Rightarrow w = 70$



Sample Group Event (1994 Sample Group Event)

SG.1 If $a*b = ab + 1$, and $s = (2*4)*2$, find s.

Reference: 1984 FG6.4

$2*4 = 2\times4 + 1 = 9$

$s = (2*4)*2 = 9*2$

$= 9\times2 + 1 = 19$

SG.2 If the n^{th} prime number is s, find n.

Reference: 1989 FSG.3, 1990 FI5.4

2, 3, 5, 7, 11, 13, 17, 19

$n = 8$

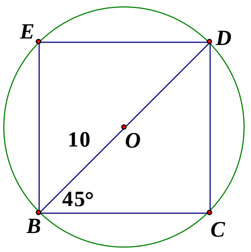
SG.3 If $K = \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\left(1-\frac{1}{50}\right)$, find K in the simplest fractional form.

Reference: 1984 FG9.1, 1986 FG10.4

$K = \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\cdots\times\frac{49}{50} = \frac{1}{50}$

SG.4 If A is the area of a square inscribed in a circle of radius 10, find A.

Reference: 1984 FG10.1, 1989 FI3.3



Let the square be $BCDE$.

$BC = 20\cos 45° = 10\sqrt{2}$

$A = \left(10\sqrt{2}\right)^2 = 200$

Group Event 6

G6.1 The average of p, q, r is 4. The average of p, q, r, x is 5. Find x.

Reference: 1986 FG6.4, 1987 FG10.1, 1988 FG9.2

$p + q + r = 12$

$p + q + r + x = 20$

$x = 8$

G6.2 A wheel of a truck travelling at 60 km/h makes 4 revolutions per second.

If its diameter is $\frac{y}{6\pi}$ m, find y.

$60 \text{ km/h} = \frac{60000}{3600} \text{ m/s} = \frac{50}{3} \text{ m/s}$

$\frac{y}{6\pi} \times \pi \times 4 = \frac{50}{3}$

$\Rightarrow y = 25$

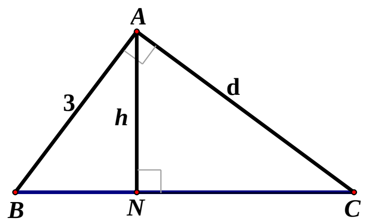
G6.3 If $\sin(55 - y)° = \frac{d}{x}$, find d.

$\sin 30° = \frac{d}{8} = \frac{1}{2}$

$\Rightarrow d = 4$

G6.4 In the figure, $BA \perp AC$ and $AN \perp BC$. If $AB = 3$, $AC = d$, $AN = h$, find h.

Reference: 1992 FI5.3



$BC^2 = 3^2 + 4^2$ (Pythagoras' theorem)

$\Rightarrow BC = 5$

Area of $\Delta ABC = \frac{1}{2} \cdot 5 \times h = \frac{1}{2} \cdot 3 \times 4$

$\Rightarrow h = \frac{12}{5}$

Group Event 7

G7.1 Let $M=\dfrac{78^3+22^3}{78^2-78\times 22+22^2}$. Find M.

Similar questions: 1984 FG6.1

$$M=\frac{78^3+22^3}{78^2-78\times 22+22^2}$$

$$=\frac{(78+22)\left(78^2-78\times 22+22^2\right)}{78^2-78\times 22+22^2}$$

$= 100$

G7.2 When the positive integer N is divided by 6, 5, 4, 3 and 2, the remainders are 5, 4, 3, 2 and 1 respectively. Find the least value of N.

Reference: 1990 HI13, 2013 FG4.3

$N + 1$ is divisible by 6, 5, 4, 3 and 2.

The L.C.M. of 6, 5, 4, 3 and 2 is 60.

$\therefore$ The least value of N is 59.

G7.3 A man travels 10 km at a speed of 4 km/h and another 10 km at a speed of 6 km/h. If the average speed of the whole journey is x km/h, find x.

$$x=\frac{20}{\frac{10}{4}+\frac{10}{6}}=\frac{24}{5}$$

G7.4 If $S = 1 + 2 - 3 - 4 + 5 + 6 - 7 - 8 + \ldots + 1985$, find S.

Reference: 1988 FG6.4, 1990 FG10.1, 1991 FSI.1, 1992 FI1.4

$S = 1 + (2 - 3 - 4 + 5) + (6 - 7 - 8 + 9) + \ldots + (1982 - 1983 - 1984 + 1985) = 1$

Group Event 8

Similar Questions 1988 FG7.1-2, 1990 FG7.3-4

M, *N* are positive integers less than 10 and 258024M8 × 9 = 2111110N × 11.

G8.1 Find *M*.

11 and 9 are relatively prime ⇒ 258024M8 is divisible by 11

⇒ 2 + 8 + 2 + *M* – (5 + 0 + 4 + 8) is divisible by 11

⇒ *M* – 5 = 11*k*

⇒ *M* = 5

G8.2 Find *N*.

2111110N is divisible by 9

⇒ 2 + 1 + 1 + 1 + 1 + 1 + *N* = 9*t*

⇒ *N* = 2

G8.3 A convex 20-sided polygon has *x* diagonals. Find *x*.
Reference: 1984 FG10.3, 1988 FG6.2, 1989 FG6.1, 1991 FI2.3, 2001 FI4.2, 2005 FI1.4

$x = C_2^{20} - 20$

$= \frac{20 \times 19}{2} - 20$

= 170

G8.4 If *y* = *ab* + *a* + *b* + 1 and *a* = 99, *b* = 49, find *y*.

Reference: 1986 FG9.3, 1988 FG6.3, 1990 FG9.2

y = (*a* + 1)(*b* + 1)

= (99 + 1)(49 + 1)

= 5000

Group Event 9

G9.1 The lengths of the 3 sides of ΔLMN are 8, 15 and 17 respectively.

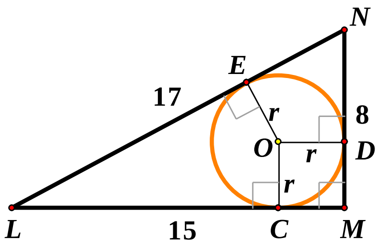
If the area of ΔLMN is A, find A.

$8^2 + 15^2 = 64 + 225 = 289 = 17^2$

$\therefore$ ΔLMN is a right-angled triangle

$A = \frac{8\times 15}{2} = 60$

G9.2 If r is the length of the radius of the circle inscribed in ΔLMN, find r.



Reference: 1989 HG9

Let O be the centre and the radius of the circle be r, which touches the triangle at C, D and E.

$OC \perp LM$, $OD \perp MN$, $OE \perp LN$ (tangent $\perp$ radius)

$ODMC$ is a rectangle (which consists of 3 right angles)

$OC = r = OD$ (radii)

$\Rightarrow$ $OCMD$ is a square.

$CM = MD = r$ (opp. sides, rectangle)

$LC = 15 - r$, $ND = 8 - r$

$LE = LC = 15 - r$, $NE = ND = 8 - r$ (tangent from ext. pt.)

$LE + NE = LN$

$\Rightarrow 15 - r + 8 - r = 17$

$\Rightarrow r = 3$

G9.3 If the r^{th} day of May in a year is Friday and the n^{th} day of May in the same year is Monday, where $15 < n < 25$, find n.

Reference: 1984 FG6.3, 1987 FG8.4, 1988 FG10.2

3^{rd} May is Friday

17^{th} May is Friday

$\Rightarrow 20^{\text{th}}$ May is Monday

$\Rightarrow n = 20$

G9.4 If the sum of the interior angles of an n-sided convex polygon is $x°$, find x.

$x = 180\times(20 - 2) = 3240$ ($\angle$s sum of polygon)

Group Event 10

G10.1 The sum of 3 consecutive odd integers (the smallest being k) is 51. Find k.

$k + k + 2 + k + 4 = 51$

$\Rightarrow k = 15$

G10.2 If $x^2 + 6x + k \equiv (x + a)^2 + C$, where a, C are constants, find C.

Reference: 1984 FI2.4, 1986 FG7.3, 1987 FSI.1, 1988 FG9.3

$x^2 + 6x + 15 \equiv (x + 3)^2 + 6$

$C = 6$

G10.3 If $\frac{p}{q} = \frac{q}{r} = \frac{r}{s} = 2$ and $R = \frac{p}{s}$, find R.

$R = \frac{p}{s}$

$= \frac{p}{q} \times \frac{q}{r} \times \frac{r}{s}$

$= 2^3 = 8$

G10.4 If $A = \frac{3^n \cdot 9^{n+1}}{27^{n-1}}$, find A.

$A = \frac{3^n \cdot 9^{n+1}}{27^{n-1}}$

$= \frac{3^n \cdot 3^{2n+2}}{3^{3n-3}}$

$= 3^6 = 243$