Individual Events

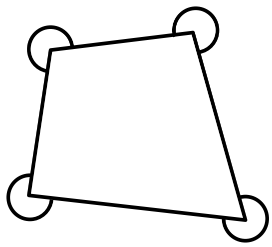
SI	***a***	1080	**I1**	***a***	6	**I2**	***h***	4	**I3**	***m***	2900	**I4**	***n***	39	**I5**	***a***	36
	b	8		***b***	2		***k***	32		***x***	8		***m***	78		***b***	48
	c	3		***c***	7		***p***	3		***y***	12		***p***	4		***p***	4
	d	64		***d***	–16		****q***	16 see the remark		***n***	33		***q***	7		***q***	6

Group Events

SG	***a***	2	**G6**	***p***	7	**G7**	***r***	2	**G8**	***A***	4	**G9**	***C***	93	**G10**	***P***	10
	b	–3		***q***	5		***s***	7		***B***	2		***n***	6		***x***	9
	p	60		***r***	–96		***a***	5		***C***	8		***S***	5000		***k***	2
	q	136		***t***	18		***p***	$\frac{1}{2}=0.5$		***D***	5		***d***	17		***S***	$\frac{11}{20}$

Sample Individual Event

SI.1 In the given figure, the sum of the four marked angles is $a°$. Find a.



Sum of interior angles of a quadrilateral = 360°

angle sum of four vertices = 4×360° = 1440°

$a = 1440 - 360 = 1080$

SI.2 The sum of the interior angles of a regular b-sided polygon is $a°$. Find b.

$180(b - 2) = 1080 = 180\times6$

$\Rightarrow b = 8$

SI.3 If $b^5 = 32^c$, find c.

$8^5 = 32^c \Rightarrow 2^{15} = 2^{5c}$

$\Rightarrow c = 3$

SI.4 If $c = \log_4 d$, find d.

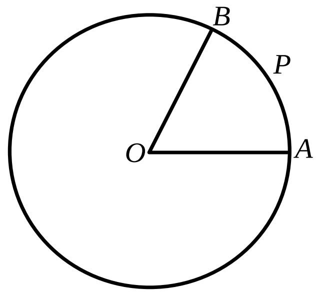
$3 = \log_4 d$

$\Rightarrow d = 4^3 = 64$

Individual Event 1

I1.1 The given figure shows a circle of radius 18 cm, centre O.

If $\angle AOB = \frac{\pi}{3}$ and the length of arc APB is $a\pi$ cm, find a.



$a\pi = 18 \times \frac{\pi}{3}$

$a = 6$

I1.2 If the solution of the inequality $2x^2 - ax + 4 < 0$ is $1 < x < b$, find b.

$2x^2 - 6x + 4 < 0$

$\Rightarrow 2(x^2 - 3x + 2) < 0$

$2(x - 1)(x - 2) < 0$

$1 < x < 2$

$\Rightarrow b = 2$

I1.3 If $b(2x - 5) + x + 3 \equiv 5x - c$, find c.

$2(2x - 5) + x + 3 \equiv 5x - c$

$\Rightarrow c = 7$

I1.4 The line through (2, 6) and (5, c) cuts the x-axis at (d, 0). Find d.

Similar question: 1985 FI2.3

They have the same slope

$\Rightarrow \frac{0-6}{d-2} = \frac{7-6}{5-2}$

$d - 2 = -18$

$d = -16$

Individual Event 2

I2.1 If the equation $3x^2 - 4x + \frac{h}{3} = 0$ has equal roots, find h.

$$\Delta = (-4)^2 - 4(3)\cdot\frac{h}{3} = 0$$

$h = 4$

I2.2 If the height of a cylinder is doubled and the new radius is h times the original, then the new volume is k times the original. Find k.

Let the original radius be r, the original height be p. Then the new radius is $4r$, the new height is $2p$.

$\pi(4r)^2(2p) = k\pi r^2\cdot p$

$k = 32$

I2.3 If $\log_{10}210 + \log_{10}k - \log_{10}56 + \log_{10}40 - \log_{10}120 + \log_{10}25 = p$, find p.

$$p = \log_{10}\frac{210\times 32\times 40\times 25}{56\times 120}$$

$= \log_{10}(10\times 4\times 25)$

$= 3$

I2.4 If $\sin A = \frac{p}{5}$ and $\frac{\cos A}{\tan A} = \frac{q}{15}$, find q.

$\sin A = \frac{3}{5}$

$$\frac{\cos A}{\tan A} = \frac{q}{15}$$

$$\frac{\cos^2 A}{\sin A} = \frac{q}{15}$$

$$\frac{1-\sin^2 A}{\sin A} = \frac{1-\left(\frac{3}{5}\right)^2}{\frac{3}{5}} = \frac{16}{15} = \frac{q}{15}$$

$q = 16$

Remark: A type-writing mistake is found in the original version:

If $\sin A = \frac{p}{3}$ and $\frac{\cos A}{\tan A} = \frac{q}{15}$, find q.

Using the given condition: $\sin A = \frac{p}{3} = 1$

We arrives at the conclusion that $A = 180°n + 90°$, where n is an integer.

So that $\frac{\cos A}{\tan A}$ will be undefined because the denominator becomes 0.

As HKMO 1990 Final Sample Individual Event is the same as HKMO 1986 Final Individual Event 2, the error is found and corrected.

Individual Event 3

I3.1 The monthly salaries of 100 employees in a company are as shown:

Sa1aries ($)	6000	4000	2500
No. of employees	5	15	80

If the mean salary is $*m*, find *m*.

$m=\frac{6000\times 5+4000\times 15+2500\times 80}{5+15+80}$

$=\frac{290000}{100}=2900$

I3.2 If $8\sin^2(m+10)° + 12\cos^2(m+25)° = x$, find x.

$x = 8\sin^2 2910° + 12\cos^2 2925°$

$= 8\sin^2(360\times 8+30)° + 12\cos^2(360\times 8+45)°$

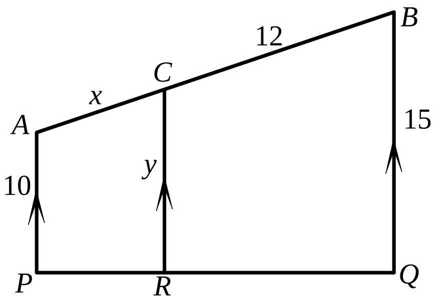
$= 8\sin^2 30° + 12\cos^2 45°$

$=8\cdot\left(\frac{1}{2}\right)^2+12\cdot\left(\frac{1}{\sqrt{2}}\right)^2$

$= 8$

Remark: A mistake was found in the solution by a student, Nicholas Ng (吳庭俊). The mistake was corrected. Thanks for Mr. Ng pointing out the mistake.

I3.3 In the figure, $AP // CR // BQ$, $AC = x$, $CB = 12$, $AP = 10$, $BQ = 15$ and $CR = y$. Find y.



Reference: 1991 FI5.3

Draw $ADE // PRQ$, cutting CR at D, BQ at E respectively.

$APRD$, $APQE$ are //-grams

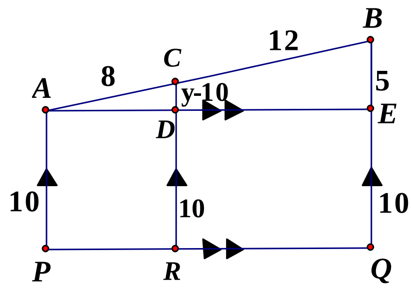
$DR = 10 = EQ$ (opp. sides, //-gram)

$CD = y - 10$, $BE = 15 - 10 = 5$

$\Delta ACD \sim \Delta ABE$ (equiangular)

$(y - 10) : 8 = 5 : 20$ (corr. of sides, ~Δ's)

$y = 12$



I3.4 Define $(a, b, c)\cdot(p, q, r) = ap + bq + cr$, where a, b, c, p, q, r are real numbers.

If $(3, 4, 5)\cdot(y, -2, 1) = n$, find n.

Reference: 1989 HI13

$n = 3y - 8 + 5$

$= 3\times 12 - 3$

$= 33$

Individual Event 4

I4.1 It is known that $\begin{cases} 1=1^2 \\ 1+3=2^2 \\ 1+3+5=3^2 \\ 1+3+5+7=4^2 \end{cases}$. If $1 + 3 + 5 + \ldots + n = 20^2$, find n.

$1 + 3 + 5 + \ldots + (2m - 1) = m^2 = 20^2$

$m = 20$

$n = 2(20) - 1 = 39$

I4.2 If the lines $x + 2y = 3$ and $nx + my = 4$ are parallel, find m.

Reference: 1987 FSG.4, 1989 FSG.2

$-\frac{1}{2} = -\frac{n}{m}$

$\Rightarrow \frac{1}{2} = \frac{39}{m}$

$m = 78$

I4.3 If a number is selected from the whole numbers 1 to m, and if each number has an equal chance of being selected, the probability that the number is a factor of m is $\frac{p}{39}$, find p.

Let S be the sample space, $n(S) = 78$

Favourable outcomes = {1, 2, 3, 6, 13, 26, 39, 78}

$\frac{p}{39} = \frac{8}{78}$

$\Rightarrow p = 4$

I4.4 A boy walks from home to school at a speed of p km/h and returns home along the same route at a speed of 3 km/h. If the average speed for the double journey is $\frac{24}{q}$ km/h, find q.

Let the distance of the single journey be x km.

$\frac{24}{q} = \frac{2x}{\frac{x}{4} + \frac{x}{3}}$

$\frac{24}{q} = \frac{24}{7}$

$\Rightarrow q = 7$

Individual Event 5

I5.1 A die is rolled. If the probability of getting a prime number is $\frac{a}{72}$, find a.

Favourable outcomes = {2, 3, 5}

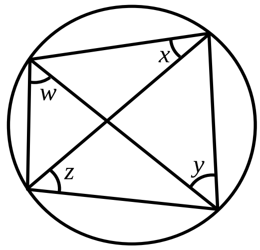
$\frac{a}{72}=\frac{3}{6}\Rightarrow a=36$

I5.2 In the figure, $x = a°$, $y = 44°$, $z = 52°$ and $w = b°$. Find b.

$x + w + y + z = 180$ (∠s in the same segment, ∠s sum of Δ)

$36 + b + 44 + 52 = 180$

$b = 48$



I5.3 A, B are two towns b km apart. Peter cycles at a speed of 7 km/h from A to B and at the same time John cycles from B to A at a speed of 5 km/h. If they meet after p hours, find p.

$p = 48\div(5 + 7) = 4$

I5.4 The base of a pyramid is a triangle with sides 3 cm, p cm and 5 cm. If the height and volume of the pyramid are q cm and 12 cm^3 respectively, find q.

$3^2 + 4^2 = 5^2 \Rightarrow$ the triangle is a right-angled triangle (Converse, Pythagoras' theorem)

Base area $=\frac{1}{2}\cdot 3\cdot 4\text{ cm}^2 = 6\text{ cm}^2$

Volume $=\frac{1}{3}\cdot 6\cdot q\text{ cm}^3 = 12\text{ cm}^3 \Rightarrow q = 6$

Sample Group Event

SG.1 The sum of two numbers is 50, and their product is 25.
If the sum of their reciprocals is a, find a.

Reference: 1983 FG6.3, 1984 FSG.1, 1985 FSI.1

Let the two numbers be x and y.

$x + y = 50$ and $xy = 25$

$a=\frac{1}{x}+\frac{1}{y}=\frac{x+y}{xy}=2$

SG.2 If the lines $ax + 2y + 1 = 0$ and $3x + by + 5 = 0$ are perpendicular, find b.

Reference: 1983 FG9.3, 1984 FSG.3, 1985 FI4.1, 1987 FG10.2, 1988 FG8.2

$-\frac{a}{2}\times\left(-\frac{b}{3}\right)=-1\Rightarrow b=-3$

SG.3 The area of an equilateral triangle is $100\sqrt{3}$ cm^2. If its perimeter is p cm, find p.

Reference: 1984FI4.4, 1985 FSI.4, 1987 FG6.2, 1988 FG9.1

Each side $=\frac{p}{3}$ cm

$\frac{1}{2}\cdot\left(\frac{p}{3}\right)^2\sin 60^\circ=100\sqrt{3}$

$p = 60$

SG.4 If $x^3 - 2x^2 + px + q$ is divisible by $x + 2$, find q.

$(-2)^3 - 2(-2)^2 + 60(-2) + q = 0$

$q = 136$

Group Event 6

G6.1 If $12345\times6789 = a\times10^p$ where p is a positive integer and $1 \le a < 10$, find p.

$12345\times6789 = 1.2345\times10^4 \times 6.789\times10^3 = a\times10^7$,

where $a = 1.2345\times6.789$

$\approx 1.2\times6.8 = 8.16$

$1 \le a < 10$

$p = 7$

G6.2 If (p, q), $(5, 3)$ and $(1, -1)$ are collinear, find q.

Reference: 1984 FSG.4, 1984 FG7.3, 1987 FG7.4, 1989 HI8

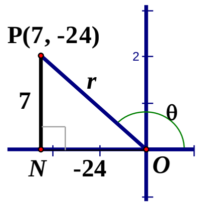
$$\frac{q-3}{7-5}=\frac{3-(-1)}{5-1}$$

$q - 3 = 2$

$\Rightarrow q = 5$

G6.3 If $\tan\theta = \frac{-7}{24}$, $90° < \theta < 180°$ and $100\cos\theta = r$, find r.

In the figure, $r^2 = 7^2 + (-24)^2$ (Pythagoras' theorem)



$r = 25$

$r = 100\cos\theta$

$=100\times\frac{-24}{25}$

$= -96$

G6.4 The average of x, y, z is 10. The average of x, y, z, t is 12. Find t.

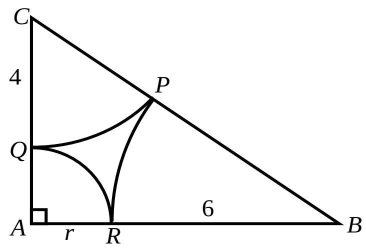
Reference: 1985 FG6.1, 1987 FG10.1, 1988 FG9.2

$x + y + z = 30$

$x + y + z + t = 48$

$t = 18$

Group Event 7



G7.1 In the figure, QR, RP, PQ are 3 arcs, centres at A, B, C respectively, touching one another at R, P, Q. If $AR = r$, $RB = 6$, $QC = 4$, $\angle A = 90°$, find r.

Reference: 1990 FG9.4

$AQ = r$, $CP = 4$, $BP = 6$

$AB = r + 6$, $AC = r + 4$, $BC = 4 + 6 = 10$

$AB^2 + AC^2 = BC^2$

$\Rightarrow (r + 6)^2 + (r + 4)^2 = 10^2$

$2r^2 + 20r - 48 = 0$

$r^2 + 10r - 24 = 0$

$(r - 2)(r + 12) = 0$

$r = 2$

G7.2 M, N are the points (3, 2) and (9, 5) respectively.

If $P(s, t)$ is a point on MN such that $MP : PN = 4 : r$, find s.

$MP : PN = 4 : 2 = 2 : 1$

$s = \frac{3\times1+9\times2}{2+1} = 7$

G7.3 $x^2 + 10x + t \equiv (x + a)^2 + k$, where t, a, k are constants. Find a.

Reference: 1984 FI2.4, 1985 FG10.2, 1987 FSI.1, 1988 FG9.3

$x^2 + 10x + t \equiv (x + 5)^2 + t - 25$

$a = 5$

G7.4 If $9^{p+2} = 240 + 9^p$, find p.

Reference: 1985 FI1.4

$81\times9^p = 240 + 9^p$

$80\times9^p = 240$

$9^p = 3$

$p = \frac{1}{2}$

Group Event 8

In the given multiplication, different letters represent different integers whose possible values are 2, 4, 5, 6, 7, 8, 9. (**Reference: 2000 HI8**) **G8.1** Find *A*. **G8.2** Find *B*. **G8.3** Find *C*. **G8.4** Find *D*.	1 A B C D E × 3 A B C D E 1
E = 7 and the carry digit is 20.	1 A B C D 7 × 3 A B C D 7 1
3D + 2 ≡ 7 (mod 10) D = 5 and the carry digit is 10	1 A B C 5 7 × 3 A B C 5 7 1
3C + 1 ≡ 5 (mod 10) C = 8 and the carry digit is 20.	1 A B 8 5 7 × 3 A B 8 5 7 1
3B + 2 ≡ 8 (mod 10) B = 2 and there is no carry digit.	1 A 2 8 5 7 × 3 A 2 8 5 7 1
3A ≡ 2 (mod 10) A = 4 and the carry digit is 10. 3 + 1 = 4 ∴ A = 4, B = 2, C = 8, D = 5	1 4 2 8 5 7 × 3 4 2 8 5 7 1

Group Event 9

G9.1 7 oranges and 5 apples cost \$13. 3 oranges and 4 apples cost \$8. 37 oranges and 45 apples cost \$$C$. Find C.

Let the cost of an orange be \$$x$ and the cost of an apple be \$$y$.

$7x + 5y = 13$ …… (1)

$3x + 4y = 8$ …… (2)

4(1) – 5(2): $13x = 12$

$\Rightarrow x = \frac{12}{13}$

7(2) – 3(1): $13y = 17$

$\Rightarrow y = \frac{17}{13}$

$C = 37x + 45y = \frac{12\times 37 + 17\times 45}{13} = 93$

G9.2 There are exactly n values of θ satisfying the equation $(\sin^2\theta - 1)(2\sin^2\theta - 1) = 0$, where $0° \le \theta \le 360°$. Find n.

$\sin\theta = \pm 1, \pm\frac{1}{\sqrt{2}}$

$\theta = 90°, 270°, 45°, 135°, 225°, 315°$

$n = 6$

G9.3 If $S = ab + a - b - 1$ and $a = 101$, $b = 49$, find S.

Reference: 1985 FG8.4, 1988 FG6.3, 1990 FG9.1

$S = (a - 1)(b + 1) = 100\times 50 = 5000$

G9.4 If d is the distance between the points (13, 5) and (5, −10), find d.

Reference: 1984 FG10.4

$d = \sqrt{(13-5)^2 + [5-(-10)]^2} = 17$

Group Event 10

G10.1 If $b + c = 3$ ⋯⋯ (1), $c + a = 6$ ⋯⋯ (2), $a + b = 7$ ⋯⋯ (3) and $P = abc$, find P.

Reference: 1989 HI15, 1990 HI7

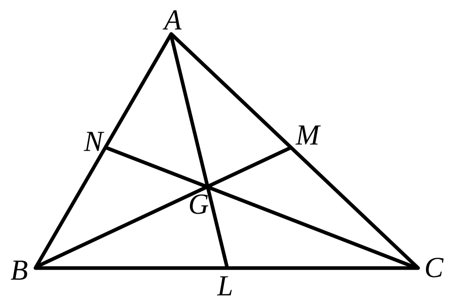
(1) + (2) – (3): $2c = 2 \Rightarrow c = 1$

(1) + (3) – (2): $2b = 4 \Rightarrow b = 2$

(2) + (3) – (1): $2a = 10 \Rightarrow a = 5$

$P = 1\times2\times5 = 10$

G10.2 The medians AL, BM, CN of ΔABC meet at G. If the area of ΔABC is 54 cm^2 and the area of ΔANG is x cm^2. Find x.



$\because BL : LC = 1 : 1$

$\therefore$ area of $\Delta ABL = \frac{1}{2}\cdot 54\text{ cm}^2 = 27\text{ cm}^2$

$\because AG : GL = 2 : 1$

$\therefore$ area of $\Delta ABG = \frac{2}{3}\cdot 27\text{cm}^2 = 18\text{ cm}^2$

$\because AN : NB = 1 : 1$

$\therefore$ area of $\Delta ANG = \frac{1}{2}\cdot 18\text{ cm}^2 = 9\text{ cm}^2$

$x = 9$

G10.3 If $k = \dfrac{3\sin\theta + 5\cos\theta}{2\sin\theta + \cos\theta}$ and $\tan\theta = 3$, find k.

Reference: 1987 FG8.1, 1989 FSG.4, 1989 FG10.3, 1990 FG7.2

$$k = \frac{(3\sin\theta + 5\cos\theta)\div\cos\theta}{(2\sin\theta+\cos\theta)\div\cos\theta}$$

$$= \frac{3\tan\theta + 5}{2\tan\theta + 1}$$

$$= \frac{3\times3+5}{2\times3+1} = 2$$

G10.4 If $S = \left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\left(1-\frac{1}{10^2}\right)$, find S.

Reference: 1999 FIS.4, 2014 FG3.1

$$S = \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\left(1-\frac{1}{10}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\cdots\left(1+\frac{1}{10}\right)$$

$$= \left(\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\cdots\times\frac{9}{10}\right)\times\left(\frac{3}{2}\times\frac{4}{3}\times\frac{5}{4}\times\cdots\times\frac{11}{10}\right)$$

$$= \frac{1}{10}\times\frac{11}{2}$$

$$= \frac{11}{20}$$