

Individual Events

SI	<i>a</i>	10	I1	<i>A</i>	380	I2	<i>r</i>	3	I3	<i>x</i>	30	I4	<i>a</i>	8	I5	<i>a</i>	1968
	<i>b</i>	280		<i>B</i>	70		<i>x</i>	1		<i>a</i>	6		<i>b</i>	17		<i>b</i>	2
	<i>c</i>	400		<i>n</i>	60		<i>y</i>	15		<i>c</i>	8		<i>d</i>	287		<i>c</i>	25
	<i>d</i>	120		<i>m</i>	5		<i>p</i>	40		<i>f(4)</i>	63		<i>K</i>	280		<i>d</i>	95

Group Events

SG	<i>A</i>	10	G6	<i>p</i>	60	G7	<i>A</i>	75	G8	<i>A</i>	2	G9	<i>A</i>	1	G10	<i>A</i>	20
	<i>B</i>	4		<i>k</i>	100		<i>B</i>	3		<i>B</i>	52		<i>B</i>	0		<i>B</i>	30
	<i>C</i>	900		<i>N</i>	9		<i>C</i>	2		<i>N</i>	5		<i>C</i>	8		<i>C</i>	2
	<i>D</i>	18		<i>M</i>	999999		<i>D</i>	5		<i>K</i>	16		<i>D</i>	9		<i>D</i>	18

Sample Individual Event

SI.1 If $x^2 - 8x + 26 \equiv (x + k)^2 + a$, find a .

Reference: 1984 FI2.4, 1985 FG10.2, 1986 FG7.3, 1988 FG9.3

$$x^2 - 8x + 26 \equiv (x - 4)^2 + 26 - 16$$

$$a = 10$$

SI.2 If $\sin a^\circ = \cos b^\circ$, where $270 < b < 360$, find b .

$$\sin 10^\circ = \cos b^\circ$$

$$\cos b^\circ = \cos 80^\circ$$

$$b = 360 - 80 = 280$$

SI.3 X sold an article to Y for $\$b$ at a loss of 30%. If the cost price of the article for X is $\$c$, find c .

$$c \cdot (1 - 30\%) = 280$$

$$c = 400$$

SI.4 In the figure, O is the centre of the circle. If $\angle ACB = \frac{3c^\circ}{10}$ and $\angle AOB$

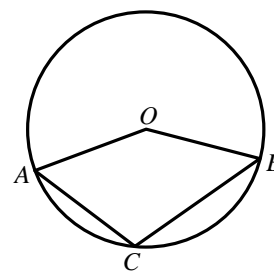
$= d^\circ$, find d .

$$\angle ACB = 120^\circ$$

$$\text{reflex } \angle AOB = 240^\circ (\angle \text{ at centre twice } \angle \text{ at } \odot^{\text{ce}})$$

$$\angle AOB = 120^\circ (\angle \text{ s at a point})$$

$$d = 120$$



Individual Event 1

I1.1 If $A = 11 + 12 + 13 + \dots + 29$, find A .

$$A = \frac{1}{2}(11 + 29) \cdot 19 = 380$$

I1.2 If $\sin A^\circ = \cos B^\circ$, where $0 < B < 90$, find B .

$$\sin 380^\circ = \cos B^\circ$$

$$\sin 20^\circ = \cos B^\circ$$

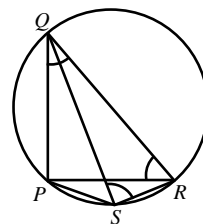
$$B = 70$$

I1.3 In the given figure, $\angle PQR = B^\circ$, $\angle PRQ = 50^\circ$. If $\angle QSR = n^\circ$, find n .

$$\angle PQR = 70^\circ$$

$$\angle QPR = 60^\circ \text{ (}\angle\text{s sum of } \Delta\text{)}$$

$$n = 60 \text{ (}\angle\text{s in the same segment)}$$



I1.4 n cards are marked from 1 to n and one is drawn at random.

If the chance of it being a multiple of 5 is $\frac{1}{m}$, find m .

Favourable outcome = $\{5, 10, \dots, 60\}$

$$\frac{1}{m} = \frac{12}{60} = \frac{1}{5}$$

$$\Rightarrow m = 5$$

Individual Event 2**12.1** The volume of a sphere with radius r is 36π , find r .

$$\frac{4\pi}{3}r^3 = 36\pi$$

$$r = 3$$

12.2 If $r^x + r^{1-x} = 4$ and $x > 0$, find x .

$$3^x + \frac{3}{3^x} = 4 \quad (\text{It is straight forward by guessing } x = 1)$$

$$(3^x)^2 - 4 \cdot 3^x + 3 = 0$$

$$(3^x - 1)(3^x - 3) = 0$$

$$3^x = 1 \text{ or } 3$$

$$x = 0 \text{ (rejected, as } x > 0) \text{ or } 1$$

12.3 In $a : b = 5 : 4$, $b : c = 3 : x$ and $a : c = y : 4$, find y .

$$a : b : c = 15 : 12 : 4$$

$$a : c = 15 : 4$$

$$\Rightarrow y = 15$$

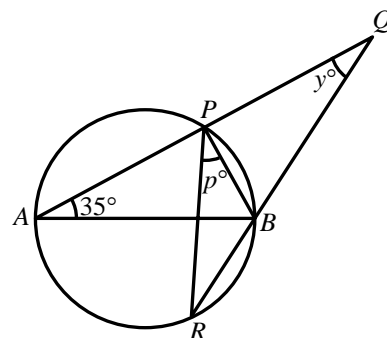
12.4 In the figure, AB is a diameter of the circle. APQ and RBQ are straight lines.If $\angle PAB = 35^\circ$, $\angle PQB = y^\circ$ and $\angle RPB = p^\circ$, find p .

$$\angle ABR = 35^\circ + y^\circ = 50^\circ \text{ (ext. } \angle \text{ of } \Delta)$$

$$\angle APR = \angle ABR = 50^\circ \text{ (}\angle\text{s in the same segment)}$$

$$p + 50 = 90 \text{ (}\angle \text{ in semi-circle)}$$

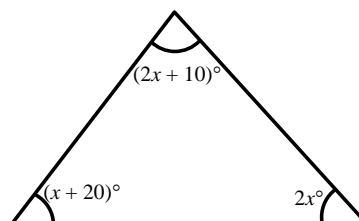
$$p = 40$$



Individual Event 3**I3.1** In the figure, find x .

$$x + 20 + 2x + 10 + 2x = 180 \text{ (}\angle\text{s sum of } \Delta\text{)}$$

$$x = 30$$

**I3.2** The coordinates of the points P and Q are $(a, 2)$ and $(x, -6)$ respectively.If the coordinates of the mid-point of PQ is $(18, b)$, find a .

$$\frac{1}{2}(a + 30) = 18$$

$$a = 6$$

I3.3 A man travels from X to Y at a uniform speed of a km/h and returns at a uniform speed of $2a$ km/h. If his average speed is c km/h, find c .Let the distance between X and Y be s km.

$$c = \frac{2s}{\frac{s}{a} + \frac{s}{2a}} = \frac{2}{\frac{1}{a} + \frac{1}{2a}} = 8$$

I3.4 If $f(y) = 2y^2 + cy - 1$, find $f(4)$.

$$f(4) = 2(4)^2 + 8(4) - 1 = 63$$

Individual Event 4**I4.1** If the curve $y = 2x^2 - 8x + a$ touches the x -axis, find a .

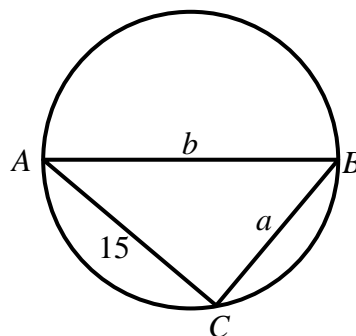
$$\Delta = (-8)^2 - 4(2)a = 0$$

$$a = 8$$

I4.2 In the figure, AB is a diameter of the circle.If $AC = 15$, $BC = a$ and $AB = b$, find b . $\angle ACB = 90^\circ$ (\angle in semi-circle)

$$b^2 = 15^2 + 8^2 \text{ (Pythagoras' theorem)}$$

$$b = 17$$

**I4.3** The line $5x + by + 2 = d$ passes through $(40, 5)$. Find d .**Reference: 1984 FI2.3**

$$d = 5(40) + 17(5) + 2 = 287$$

I4.4 X sold an article to Y for $\$d$ at a profit of 2.5%.If the cost price of the article for X is $\$K$, find K .

$$K = 287 \div (1 + 2.5\%) = 280$$

Individual Event 5

15.1 Let $x = 19.\dot{8}\dot{7}$. If $19.\dot{8}\dot{7} = \frac{a}{99}$, find a . (Hint: $99x = 100x - x$)

$$99x = 100x - x = 1987 + 0.\dot{8}\dot{7} - 19.\dot{8}\dot{7} = 1968$$

$$x = \frac{1968}{99}$$

$$\Rightarrow a = 1968$$

15.2 If $f(y) = 4 \sin y^\circ$ and $f(a - 18) = b$, find b .

$$b = f(a - 18) = f(1950)$$

$$= 4 \sin 1950^\circ$$

$$= 4 \sin(360^\circ \times 5 + 150^\circ)$$

$$= 4 \sin 150^\circ = 2$$

15.3 If $\frac{\sqrt{3}}{b\sqrt{7}-\sqrt{3}} = \frac{2\sqrt{21}+3}{c}$, find c .

$$\frac{\sqrt{3}}{2\sqrt{7}-\sqrt{3}} \cdot \frac{2\sqrt{7}+\sqrt{3}}{2\sqrt{7}+\sqrt{3}} = \frac{2\sqrt{21}+3}{c}$$

$$c = 4(7) - 3 = 25$$

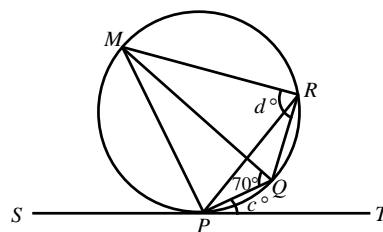
15.4 In the figure, ST is a tangent to the circle at P .

If $\angle MQP = 70^\circ$, $\angle QPT = c^\circ$ and $\angle MRQ = d^\circ$, find d .

$\angle MRP = 70^\circ$ (\angle s in the same segment)

$\angle PRQ = c^\circ = 25^\circ$ (\angle in alt. segment)

$$d = 70 + 25 = 95$$



Sample Group Event**SG.1** If $100A = 35^2 - 15^2$, find A .**Reference: 1984 FI1.1**

$$100A = (35 - 15)(35 + 15) = 1000$$

$$A = 10$$

SG.2 If $(A - 1)^6 = 27^B$, find B .

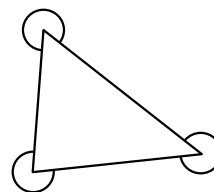
$$(10 - 1)^6 = 27^B$$

$$3^{12} = 3^{3B}$$

$$\Rightarrow B = 4$$

SG.3 In the given diagram, the sum of the three marked angles is C° . Find C .**Reference: 1984 FSI.1, 1989 FSI.1**Sum of interior angles of a triangle = 180° angle sum of three vertices = $3 \times 360^\circ = 1080^\circ$

$$C = 1080 - 180 = 900$$

**SG.4** If the lines $x + 2y + 1 = 0$ and $9x + Dy + 1 = 0$ are parallel, find D .**Reference: 1986 FI4.2, 1989 FSG.2**

$$-\frac{1}{2} = -\frac{9}{D}$$

$$\Rightarrow D = 18$$

Group Event 6**G6.1** If α, β are the roots of $x^2 - 10x + 20 = 0$, and $p = \alpha^2 + \beta^2$, find p .

$$\alpha + \beta = 10, \alpha\beta = 20$$

$$\begin{aligned} p &= (\alpha + \beta)^2 - 2\alpha\beta \\ &= 10^2 - 2(20) = 60 \end{aligned}$$

G6.2 The perimeter of an equilateral triangle is p . If its area is $k\sqrt{3}$, find k .**Reference: 1984FI4.4, 1985 FSI.4, 1986 FSG.3, 1988 FG9.1**

Length of one side = 20

$$\frac{1}{2} \cdot 20^2 \sin 60^\circ = k\sqrt{3}$$

$$k = 100$$

G6.3 Each interior angle of an N -sided regular polygon is 140° . Find N .**Reference: 1997 FI4.1**Each exterior angle = 40° (adj. \angle s on st. line)

$$\frac{360^\circ}{N} = 40^\circ \quad (\text{sum of ext. } \angle\text{s of polygon})$$

$$\Rightarrow N = 9$$

G6.4 If $M = (10^2 + 10 \times 1 + 1^2)(10^2 - 1^2)(10^2 - 10 \times 1 + 1^2)$, find M .

$$\begin{aligned} M &= (10^2 + 10 \times 1 + 1^2)(10 - 1)(10 + 1)(10^2 - 10 \times 1 + 1^2) \\ &= (10^3 - 1)(10^3 + 1) \\ &= 10^6 - 1 = 999999 \end{aligned}$$

Group Event 7**G7.1** The acute angle formed by the hands of a clock at 3:30 p.m. is A° . Find A .**Reference** 1984 FG7.1, 1985 FI3.1, 1989 FI1.1, 1990 FG6.3, 2007 HI1At 3:00 p.m., the angle between the arms of the clock = 90° From 3:00 p.m. to 3:30 p.m., the hour-hand had moved $360^\circ \times \frac{1}{12} \times \frac{1}{2} = 15^\circ$.The minute hand had moved 180° .

$$A = 180 - 90 - 15 = 75$$

G7.2 If $\tan(3A + 15)^\circ = \sqrt{B}$, find B .

$$\tan(225 + 15)^\circ = \sqrt{B}$$

$$\Rightarrow B = 3$$

G7.3 If $\log_{10} AB = C \log_{10} 15$, find C .

$$\log_{10} (75 \times 3) = C \log_{10} 15$$

$$\log_{10} 225 = C \log_{10} 15$$

$$\Rightarrow C = 2$$

G7.4 The points $(1, 3)$, $(4, 9)$ and $(2, D)$ are collinear. Find D .**Reference:** 1984 FSG.4, 1984 FG7.3, 1986 FG6.2, 1989 HI8

$$\frac{D-9}{2-4} = \frac{9-3}{4-1}$$

$$D-9 = -4$$

$$\Rightarrow D = 5$$

Group Event 8

G8.1 If $A = \frac{5\sin\theta + 4\cos\theta}{3\sin\theta + \cos\theta}$ and $\tan\theta = 2$, find A .

Reference: 1986 FG10.3, 1989 FSG.4, 1989 FG10.3, 1990 FG7.2

$$\begin{aligned} A &= \frac{(5\sin\theta + 4\cos\theta) \div \cos\theta}{(3\sin\theta + \cos\theta) \div \cos\theta} \\ &= \frac{5\tan\theta + 4}{3\tan\theta + 1} \\ &= \frac{5(2) + 4}{3(2) + 1} = 2 \end{aligned}$$

G8.2 If $x + \frac{1}{x} = 2A$, and $x^3 + \frac{1}{x^3} = B$, find B .

Reference: 1983 FG7.3, 1984 FG10.2, 1985 FI1.2, 1989 HI1, 1990 HI12, 2002 FG2.2

$$\begin{aligned} x + \frac{1}{x} &= 4 \\ \Rightarrow x^2 + \frac{1}{x^2} &= 4^2 - 2 = 14 \\ B = x^3 + \frac{1}{x^3} \\ &= \left(x + \frac{1}{x}\right)\left(x^2 - 1 + \frac{1}{x^2}\right) \\ &= 4(14 - 1) = 52 \end{aligned}$$

G8.3 There are exactly N values of α satisfying the equation $\cos^3\alpha - \cos\alpha = 0$, where $0^\circ \leq \alpha \leq 360^\circ$. Find N .

$$\begin{aligned} \cos\alpha(\cos\alpha + 1)(\cos\alpha - 1) &= 0 \\ \cos\alpha &= 0, -1 \text{ or } 1 \\ \alpha &= 90, 270, 180, 0, 360 \\ \Rightarrow N &= 5 \end{aligned}$$

G8.4 If the N^{th} day of May in a year is Thursday and the K^{th} day of May in the same year is Monday, where $10 < K < 20$, find K .

Reference: 1984 FG6.3, 1985 FG9.3, 1988 FG10.2

$$\begin{aligned} 5^{\text{th}} \text{ May is Thursday} \\ 9^{\text{th}} \text{ May is Monday} \\ 16^{\text{th}} \text{ May is Monday} \\ \Rightarrow K &= 16 \end{aligned}$$

Group Event 9

In the given multiplication, different letters represent different integers ranging from 0 to 9.

$$\begin{array}{r}
 \begin{array}{cccc}
 & A & B & C & D \\
 \times & & & & 9 \\
 \hline
 & D & C & B & A \\
 & 1 & B & C & 9 \\
 \times & & & & 9 \\
 \hline
 & 9 & C & B & 1
 \end{array}
 \end{array}$$

G9.1 Find A .**G9.2** Find B .**G9.3** Find C .**G9.4** Find D .**Reference: 1994 HI6**

As there is no carry digit in the thousands digit multiplication, $A = 1$, $D = 9$

Consider the tens digit: $9C + 8 \equiv B \pmod{10}$ (1)

As there is no carry digit in the thousands digit, let the carry digit in the hundreds digit be x .

$9B + x = C$ and B, C are distinct integers different from 1 and 9

$\Rightarrow B = 0, C = x$

Sub. $B = 0$ into (1): $9C + 8 \equiv 0 \pmod{10}$

$\Rightarrow 9C \equiv 2 \pmod{10}$

$\Rightarrow C = 8$

$\therefore A = 1, B = 0, C = 8, D = 9$

Group Event 10

G10.1 The average of p, q, r and s is 5. The average of p, q, r, s and A is 8. Find A .

Reference: 1985 FG6.1, 1986 FG6.4, 1988 FG9.2

$$p + q + r + s = 20$$

$$p + q + r + s + A = 40$$

$$A = 20$$

G10.2 If the lines $3x - 2y + 1 = 0$ and $Ax + By + 1 = 0$ are perpendicular, find B .

Reference: 1983 FG9.3, 1984 FSG.3, 1985 FI4.1, 1986 FSG.2, 1988 FG8.2

$$\frac{3}{2} \times \left(-\frac{20}{B} \right) = -1 \Rightarrow B = 30$$

G10.3 When $Cx^3 - 3x^2 + x - 1$ is divided by $x + 1$, the remainder is -7 . Find C .

$$C(-1) - 3 - 1 - 1 = -7$$

$$C = 2$$

G10.4 If P, Q are positive integers such that $P + Q + PQ = 90$ and $D = P + Q$, find D .

(Hint: Factorise $1 + P + Q + PQ$)

Reference: 2002 HG9, 2012 FI4.2

WLOG assume $P \leq Q$, $1 + P + Q + PQ = 91$

$$(1 + P)(1 + Q) = 1 \times 91 = 7 \times 13$$

$$1 + P = 1 \Rightarrow P = 0 \text{ (rejected)}$$

$$\text{or } 1 + P = 7 \Rightarrow P = 6$$

$$1 + Q = 13 \Rightarrow Q = 12$$

$$D = 6 + 12 = 18$$