Individual Events

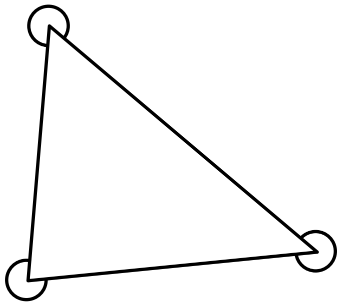
SI	***a***	900	**I1**	***P***	100	**I2**	***k***	4	**I3**	***h***	3	**I4**	***a***	18	**I5**	***a***	495
	b	7		***Q***	8		***m***	58		***k***	6		***r***	3		***b***	2
	p	2		***R***	50		***a***	2		***m***	4		***M***	$\frac{9}{4}$		***x***	99
	q	9		***S***	3		***b***	3		***p***	$\frac{15}{16}$		***w***	1		***Y***	109

Group Events

SG	***p***	75	**G6**	***x***	125	**G7**	***M***	5	**G8**	***S***	27	**G9**	***p***	60	**G10**	***n***	18
	q	0.5		***n***	10		***N***	6		***T***	135		***t***	10		***k***	22
	a	9		***y***	1000		***a***	8		***A***	9		***K***	43		***t***	96
	m	14		***K***	1003		***k***	4		***B***	0		***C***	9		***h***	$\frac{168}{25}$

Sample Individual Event (1984 Sample Individual Event)

SI.1 In the given diagram, the sum of the three marked angles is $a°$. Find a.



Sum of interior angles of a triangle = 180°

angle sum of three vertices = 3×360° = 1080°

$a = 1080 - 180 = 900$

SI.2 The sum of the interior angles of a regular b-sided polygon is $a°$. Find b.

$a = 900 = 180\times(b - 2)$

$b = 7$

SI.3 If $8^b = p^{21}$, find p.

$8^7 = p^{21}$

$2^{21} = p^{21}$

$\Rightarrow p = 2$

SI.4 If $p = \log_q 81$, find q.

$2 = p = \log_q 81$ and $q > 0$

$q^2 = 81$

$\Rightarrow q = 9$

Individual Event 1

I1.1 If $N(t) = 100\times18^t$ and $P = N(0)$, find P.

$P = 100\times18^0 = 100$

I1.2 A fox ate P grapes in 5 days, each day eating 6 more than on the previous day. If he ate Q grapes on the first day, find Q.

$Q + (Q + 6) + (Q + 12) + (Q + 18) + (Q + 24) = P = 100$

$5Q + 60 = 100$

$\Rightarrow Q = 8$

I1.3 If Q% of $\frac{25}{32}$ is $\frac{1}{Q}$% of R, find R.

$\frac{25}{32}\times\frac{8}{100} = R\times\frac{1}{100\times8}$

$\Rightarrow R = 50$

I1.4 If one root of the equation $3x^2 - ax + R = 0$ is $\frac{50}{9}$ and the other root is S, find S.

$\frac{50}{9}\times S = \text{product of roots} = \frac{R}{3} = \frac{50}{3}$

$\Rightarrow S = 3$

Individual Event 2

I2.1 If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$ and $\begin{vmatrix} 3 & 4 \\ 2 & k \end{vmatrix} = k$, find k.

$3k - 8 = k$

$\Rightarrow k = 4$

I2.2 If $50m = 54^2 - k^2$, find m.

Reference: 1984 FI1.1, 1987 FSG.1

$50m = 54^2 - 4^2 = (54 + 4)(54 - 4) = 58\times50$

$\Rightarrow m = 58$

I2.3 If $(m + 6)^a = 2^{12}$, find a.

$(58 + 6)^a = 2^{12}$

$\Rightarrow 64^a = 2^{12}$

$\Rightarrow 2^{6a} = 2^{12}$

$\Rightarrow a = 2$

I2.4 A, B and C are the points $(a, 5)$, $(2, 3)$ and $(4, b)$ respectively. If $AB \perp BC$, find b.

$A(2, 5)$, $B(2, 3)$, $C(4, b)$.

AB is parallel to y-axis

$\Rightarrow BC$ is parallel to x-axis

$\Rightarrow b = 3$

Individual Event 3

I3.1 If $\frac{\sqrt{3}}{2\sqrt{7}-\sqrt{3}}=\frac{2\sqrt{21}+h}{25}$, find h.

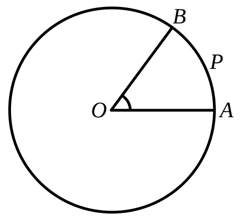
$\frac{\sqrt{3}}{2\sqrt{7}-\sqrt{3}}\cdot\frac{2\sqrt{7}+\sqrt{3}}{2\sqrt{7}+\sqrt{3}}=\frac{2\sqrt{21}+h}{25}$

$2\sqrt{21}+3=2\sqrt{21}+h$

$\Rightarrow h = 3$

I3.2 The given figure shows a circle of radius $2h$ cm, centre O.

If $\angle AOB=\frac{\pi}{3}$, and the area of sector $AOBP$ is $k\pi$ cm^2, find k.



$\frac{1}{2}\cdot(2\cdot 3)^2\cdot\frac{\pi}{3}=k\pi$

$k = 6$

I3.3 A can do a job in k days, B can do the same job in $(k + 6)$ days.

If they work together, they can finish the job in m days. Find m.

$\frac{1}{m}=\frac{1}{k}+\frac{1}{k+6}$

$\Rightarrow\frac{1}{m}=\frac{1}{6}+\frac{1}{12}$

$\Rightarrow m = 4$

I3.4 m coins are tossed. If the probability of obtaining at least one head is p, find p.

P(at least one head) = 1 – P(all tail)

$=1-\left(\frac{1}{2}\right)^4=\frac{15}{16}$

Individual Event 4

I4.1 If $f(t) = 2-\frac{t}{3}$, and $f(a) = -4$, find a.

$f(a) = 2-\frac{a}{3}=-4$

$\Rightarrow a = 18$

I4.2 If $a + 9 = 12Q + r$, where Q, r are integers and $0 < r < 12$, find r.

$18 + 9 = 27 = 12\times 2 + 3 = 12Q + r$

$r = 3$

I4.3 x, y are real numbers. If $x + y = r$ and M is the maximum value of xy, find M.

Reference: 1985 FI3.4

$x + y = 3$

$\Rightarrow y = 3 - x$

$xy = x(3 - x) = 3x - x^2 = -(x - 1.5)^2 + 2.25$

$M = 2.25 = \frac{9}{4}$

I4.4 If w is a real number and $2^{2w} - 2^w - \frac{8}{9}M = 0$, find w.

$2^{2w}-2^w-\frac{8}{9}\cdot\frac{9}{4}=0$

$\Rightarrow (2^w)^2 - 2^w - 2 = 0$

$(2^w + 1)(2^w - 2) = 0$

$\Rightarrow w = 1$

Individual Event 5

I5.1 If $0.3\dot{5}\dot{7} = \frac{177}{a}$, find a.

$$0.3\dot{5}\dot{7} = \frac{3}{10} + 0.0\dot{5}\dot{7}$$
$$= \frac{3}{10} + \frac{57}{990}$$
$$= \frac{297+57}{990}$$
$$= \frac{354}{990} = \frac{59}{165} = \frac{177}{495}$$
$$= \frac{177}{a}$$

$a = 495$

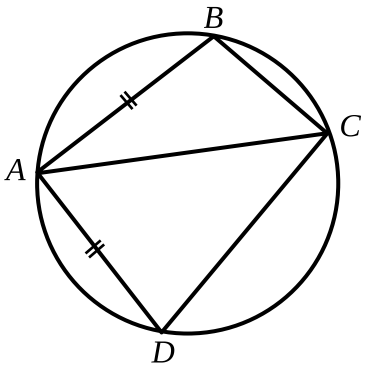
I5.2 If $\tan^2 a° + 1 = b$, find b.

$b = \tan^2 495° + 1$

$= \tan^2(180°\times3 - 45°) + 1$

$= 1 + 1 = 2$

I5.3 In the figure, $AB = AD$, $\angle BAC = 26° + b°$, $\angle BCD = 106°$. If $\angle ABC = x°$, find x.



$\angle BCA = \angle DCA = \frac{1}{2}\angle BCD = 53°$ (eq. chords eq. $\angle$s)

$\angle BAC = 28°$

$x° = \angle ABC = 180° - 28° - 53° = 99°$ ($\angle$s sum of Δ)

$x = 99$

I5.4 If $\begin{pmatrix} h & k \end{pmatrix}\begin{pmatrix} m & p \\ n & q \end{pmatrix} = \begin{pmatrix} hm+kn & hp+kq \end{pmatrix}$ and $\begin{pmatrix} 1 & 2 \end{pmatrix}\begin{pmatrix} 3 & x \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 11 & Y \end{pmatrix}$, find Y.

$\begin{pmatrix} 1\times3+2\times4 & x+2\times5 \end{pmatrix} = \begin{pmatrix} 11 & Y \end{pmatrix}$

$\Rightarrow Y = 109$

Sample Group Event (1984 Group Event 7)

SG.1 The acute angle between the 2 hands of a clock at 3:30 p.m. is $p°$. Find p.

At 3:00 p.m., the angle between the arms of the clock = 90°

From 3:00 p.m. to 3:30 p.m., the hour-hand had moved $360°\times\frac{1}{12}\times\frac{1}{2}=15°$.

The minute hand had moved 180°.

$p = 180 - 90 - 15 = 75$

SG.2 In ΔABC, $\angle B = \angle C = p°$. If $q = \sin A$, find q.

$\angle B = \angle C = 75°$, $\angle A = 180° - 75° - 75° = 30°$

$q = \sin 30° = \frac{1}{2}$

SG.3 The 3 points (1, 3), (2, 5), (4, a) are collinear. Find a.

$\frac{9-5}{4-2}=\frac{a-3}{4-1}=2$

$\Rightarrow a = 9$

SG.4 The average of 7, 9, x, y,17 is 10. If the average of x +3, x +5, y + 2, 8 and y + 18 is m, find m.

$\frac{7+9+x+y+17}{5}=10$

$\Rightarrow x + y = 17$

$m=\frac{x+3+x+5+y+2+8+y+18}{5}$

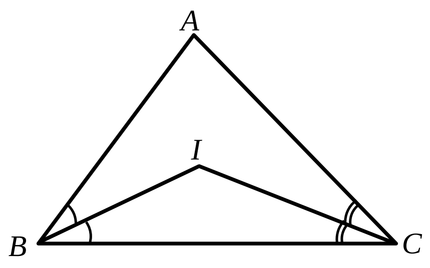
$=\frac{2(x+y)+36}{5}$

$= 14$

Group Event 6

G6.1 In the figure, the bisectors of $\angle B$ and $\angle C$ meet at I.

If $\angle A = 70°$ and $\angle BIC = x°$, find x.



Let $\angle ABI = b = \angle CBI$, $\angle ACI = c = \angle BCI$

$2b + 2c + 70 = 180$ ($\angle$s sum of Δ)

$b + c = 55$

In ΔBCI, $b + c + x = 180$ ($\angle$s sum of Δ)

$x = 180 - 55 = 125$

G6.2 A convex n-sided polygon has 35 diagonals. Find n.

Reference: 1984 FG10.3, 1985 FG8.3, 1989 FG6.1, 1991 FI2.3, 2001 FI4.2, 2005 FI1.4

$C_2^n - n = 35$

$\Rightarrow \frac{n(n-3)}{2} = 35$

$n^2 - 3n - 70 = 0$

$\Rightarrow (n - 10)(n + 7) = 0$

$n = 10$

G6.3 If $y = ab - a + b - 1$ and $a = 49$, $b = 21$, find y.

Reference: 1985 FG8.4, 1986 FG9.3, 1990 FG9.1

$y = (a + 1)(b - 1) = (49 + 1)(21 - 1) = 50\times20 = 1000$

G6.4 If $K = 1 + 2 - 3 - 4 + 5 + 6 - 7 - 8 + \dots + 1001 + 1002$, find K.

Reference: 1985 FG7.4, 1990 FG10.1, 1991 FSI.1, 1992 FI1.4

$K = 1 + (2 - 3 - 4 + 5) + (6 - 7 - 8 + 9) + \dots + (998 - 999 - 1000 + 1001) + 1002 = 1003$

Group Event 7 (**Similar Questions 1985 FG8.1-2, 1990 FG7.3-4**)

M, N are positive integers less than 10 and $8M420852 \times 9 = N9889788 \times 11$.

G7.1 Find M.

11 and 9 are relatively prime

$\Rightarrow 8M420852$ is divisible by 11

$\Rightarrow 8 + 4 + 0 + 5 - (M + 2 + 8 + 2)$ is divisible by 11

$\Rightarrow 5 - M = 11k$

$\Rightarrow M = 5$

G7.2 Find N.

$N9889788$ is divisible by 9

$\Rightarrow N + 9 + 8 + 8 + 9 + 7 + 8 + 8 = 9t$

$\Rightarrow N = 6$

G7.3 The equation of the line through (4, 3) and (12, –3) is $\frac{x}{a}+\frac{y}{b}=1$. Find a.

$\frac{y-3}{x-4}=\frac{3-(-3)}{4-12}$

$3x - 12 + 4y - 12 = 0$

$\Rightarrow 3x + 4y = 24$

$\frac{x}{8}+\frac{y}{6}=1$

$\Rightarrow a = 8$

G7.4 If $x + k$ is a factor of $3x^2 + 14x + a$, find k. (k is an integer.)

$3(-k)^2 + 14(-k) + 8 = 0$

$\Rightarrow 3k^2 - 14k + 8 = 0$

$(3k - 2)(k - 4) = 0$

$\Rightarrow k = 4$ (reject $\frac{2}{3}$)

Group Event 8

G8.1 If $\log_9 S = \frac{3}{2}$, find S.

$S = 9^{3/2} = 27$

G8.2 If the lines $x + 5y = 0$ and $Tx - Sy = 0$ are perpendicular to each other, find T.

Reference: 1983 FG9.3, 1984 FSG.3, 1985 FI4.1, 1986 FSG.2, 1987 FG10.2

$-\frac{1}{5} \times \frac{T}{27} = -1$

$T = 135$

The 3-digit number AAA, where $A \neq 0$, and the 6-digit number $AAABBB$ satisfy the following equality: $AAA \times AAA + AAA = AAABBB$.

G8.3 Find A.

$A(111) \times A(111) + A(111) = A(111000) + B(111)$

$111A^2 + A = 1000A + B$

Consider the thousands digit: $9 < A^2 \leq 81$

$\Rightarrow A = 4, 5, 6, 7, 8, 9$

When $A = 4$: $111\times16 + 4 = 4000 + B$ (rejected)

When $A = 5$: $111\times25 + 5 = 5000 + B$ (rejected)

When $A = 6$: $111\times36 + 6 = 6000 + B$ (rejected)

When $A = 7$: $111\times49 + 7 = 7000 + B$ (rejected)

When $A = 8$: $111\times64 + 8 = 8000 + B$ (rejected)

When $A = 9$: $111\times81 + 9 = 9000 + B$

$\therefore A = 9$

G8.4 Find B.

$B = 0$

Group Event 9

G9.1 The area of an equilateral triangle is $50\sqrt{12}$. If its perimeter is p, find p.

Reference: 1984FI4.4, 1985 FSI.4, 1986 FSG.3, 1987 FG6.2

Each side = $\frac{p}{3}$

$\frac{1}{2}\cdot\left(\frac{p}{3}\right)^2\sin 60^\circ = 50\sqrt{12} = 100\sqrt{3}$

$p = 60$

G9.2 The average of q, y, z is 14. The average of q, y, z, t is 13. Find t.

Reference: 1985 FG6.1, 1986 FG6.4, 1987 FG10.1

$\frac{q+y+z}{3} = 14$

$\Rightarrow q + y + z = 42$

$\frac{q+y+z+t}{4} = 13$

$\Rightarrow \frac{42+t}{4} = 13$

$t = 10$

G9.3 If $7 - 24x - 4x^2 \equiv K + A(x + B)^2$, where K, A, B are constants, find K.

Reference: 1984 FI2.4, 1985 FG10.2, 1986 FG7.3, 1987 FSI.1

$7 - 24x - 4x^2 \equiv -4(x^2 + 6x) + 7 \equiv -4(x + 3)^2 + 43$

$K = 43$

G9.4 If $C = \frac{3^{4n}\cdot 9^{n+4}}{27^{2n+2}}$, find C.

$C = \frac{3^{4n}\cdot 3^{2n+8}}{3^{6n+6}} = 9$

Group Event 10

G10.1 Each interior angle of an n-sided regular polygon is 160°. Find n.

Each exterior angle = 20° (adj. ∠s on st. line)

$$\frac{360^\circ}{n} = 20^\circ$$

$\Rightarrow n = 18$

G10.2 The n^{th} day of May in a year is Friday. The k^{th} day of May in the same year is Tuesday, where $20 < k < 26$. Find k.

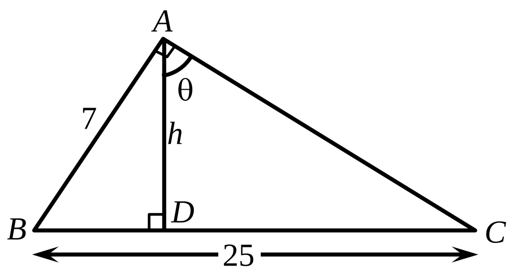
Reference: 1984 FG6.3, 1985 FG9.3, 1987 FG8.4

18^{th} May is Friday

22^{nd} May is Tuesday

$\Rightarrow k = 22$

In the figure, $AD \perp BC$, $BA \perp CA$, $AB = 7$, $BC = 25$, $AD = h$ and $\angle CAD = \theta$.



G10.3 If $100 \sin \theta = t$, find t.

$AC^2 + 7^2 = 25^2$ (Pythagoras' theorem)

$AC = 24$

$\angle ACD = 90^\circ - \theta$ (∠s sum of Δ)

$\angle ABC = \theta$ (∠s sum of Δ)

$$t = 100 \sin \theta = 100 \times \frac{24}{25} = 96$$

G10.4 Find h.

$$\text{Area of } \Delta ABC = \frac{1}{2} \cdot 7 \times 24 = \frac{1}{2} \cdot 25h$$

$$h = \frac{168}{25}$$

Method 2

In ΔABD,

$h = AB \sin \theta$

$$= 7 \times \frac{24}{25} = \frac{168}{25}$$