**Individual Events**

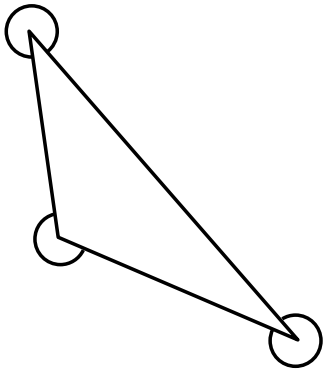
| **SI** | ***a*** | 900 | **I1** | ***a*** | 35 | **I2** | ***a*** | 7 | **I3** | **α** | 8 | **I4** | ***t*** | 13 | **I5** | ***a*** | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ***b*** | 7 | | ***b*** | 7 | | ***b*** | 3 | | ***b*** | 16 | | ***s*** | 4 | | ***b*** | 150 |
| | ***c*** | 3 | | ***c*** | 10 | | ***c*** | 9 | | ***A*** | 128 | | ***a*** | 3 | | ***n*** | 12 |
| | ***d*** | 5 | | ***d*** | 2 | | ***d*** | 5 | | ***d*** | 7 | | ***c*** | 12 | | ***k*** | 24 |

**Group Events**

| **SG** | ***a*** | 2 | **G6** | ***n*** | 8 | **G7** | ***G*** | 1 | **G8** | ***y*** | 7 | **G9** | ***x*** | 40 | **G10** | ***a*** | 6 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ***b*** | 9 | | ***k*** | 5 | | ***D*** | 8 | | ***k*** | –96 | | ***y*** | 3 | | ***x*** | 3 |
| | ***p*** | 23 | | ***u*** | 35 | | ***L*** | 2 | | ***a*** | 1 | | ***k*** | 8 | | ***k*** | 2 |
| | ***k*** | 3 | | ***a*** | 1 | | ***E*** | 5 | | ***m*** | 2 | | ***r*** | 5 | | ***y*** | 4 |

**Sample Individual Event**

**SI.1** In the given diagram, the sum of the three marked angles is $a°$. Find the value of $a$.



**Reference: 1984 FSI.1 1987 FSG.3**

Sum of interior angles of a triangle = 180°

angle sum of three vertices = 3×360° = 1080°

$a$ = 1080 – 180 = 900

**SI.2** The sum of the interior angles of a convex $b$-sided polygon is $a°$. Find the value of $b$.

**Reference 1984 FSI.2**

$a = 900 = 180\times(b-2)$

$b = 7$

**SI.3** If $27^{b-1} = c^{18}$, find the value of $c$.

$3^{3(7-1)} = c^{18}$

$c = 3$

**SI.4** If $c = \log_d 125$, find the value of $d$.

$3 = c = \log_d 125$

$d^3 = 125$

$d = 5$

**Individual Event 1**

**I1.1** The obtuse angle formed by the hands of a clock at 10:30 is $(100 + a)°$. Find the value of $a$.

**Reference 1984 FG7.1, 1985 FI3.1, 1987 FG7.1, 1990 FG6.3, 2007 HI1**

At 10:00, the angle between the arms of the clock = 60°

From 10:00 to 10:30, the hour-hand had moved $360° \times \frac{1}{12} \times \frac{1}{2} = 15°$.

The minute hand had moved 180°.

$100 + a = 180 - 60 + 15 = 135 \Rightarrow a = 35$

**I1.2** The lines $ax + by = 0$ and $x - 5y + 1 = 0$ are perpendicular to each other. Find the value of $b$.

$-\frac{35}{b} \times \frac{1}{5} = -1$

$\Rightarrow b = 7$

**I1.3** If $(b + 1)^4 = 2^{c+2}$, find the value of $c$.

$8^4 = 2^{c+2}$

$2^{3(4)} = 2^{c+2}$

$\Rightarrow c = 10$

**I1.4** If $c - 9 = \log_c (6d - 2)$, find the value of $d$.

$10 - 9 = 1 = \log_{10} (6d - 2)$

$\Rightarrow 6d - 2 = 10$

$\Rightarrow d = 2$

**Individual Event 2**

**I2.1** If $1000a = 85^2 - 15^2$, find the value of $a$.

$1000a = (85 + 15)(85 - 15) = 100 \times 70$

$\Rightarrow a = 7$

**I2.2** The point $(a, b)$ lies on the line $5x + 2y = 41$. Find the value of $b$.

$5(7) + 2b = 41$

$\Rightarrow b = 3$

**I2.3** $x + b$ is a factor of $x^2 + 6x + c$. Find the value of $c$.

Put $x = -3$ into $x^2 + 6x + c = 0$

$(-3)^2 + 6(-3) + c = 0$

$\Rightarrow c = 9$

**I2.4** If $d$ is the distance between the points $(c, 1)$ and $(5, 4)$, find the value of $d$.

$d^2 = (9 - 5)^2 + (1 - 4)^2 = 25$

$\Rightarrow d = 5$

**Individual Event 3**

**I3.1** If $\alpha + \beta = 11$, $\alpha\beta = 24$ and $\alpha > \beta$, find the value of $\alpha$.

$\alpha$ and $\beta$ are the roots of the equation $x^2 - 11x + 24 = 0$

$(x - 3)(x - 8) = 0$

$\because \alpha > \beta$

$\therefore \alpha = 8$

**I3.2** If $\tan\theta = \frac{-\alpha}{15}$, $90° < \theta < 180°$ and $\sin\theta = \frac{b}{34}$, find the value of $b$.

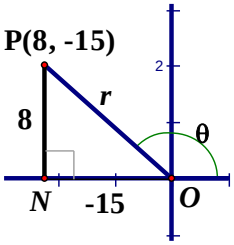
In the figure, $P = (8, -15)$

$r^2 = 8^2 + (-15)^2$ (Pythagoras' theorem)

$r = 17$

$\sin\theta = \frac{8}{17} = \frac{16}{34}$

$b = 16$



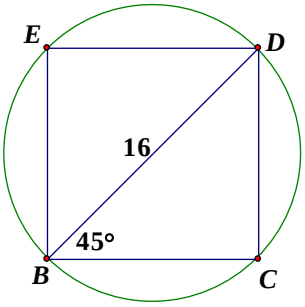

**I3.3** If $A$ is the area of a square inscribed in a circle of diameter $b$, find the value of $A$.

**Reference: 1984 FG10.1, 1985 FSG.4**

Let the square be $BCDE$.

$BC = 16\cos 45° = 8\sqrt{2}$

$A = \left(8\sqrt{2}\right)^2 = 128$




**I3.4** If $x^2 + 22x + A \equiv (x + k)^2 + d$, where $k$, $d$ are constants, find the value of $d$.

$x^2 + 22x + 128 \equiv (x + 11)^2 + 7$

$d = 7$

**Individual Event 4**

**I4.1** The average of $p, q, r$ is 12. The average of $p, q, r, t, 2t$ is 15. Find the value of $t$.

$p + q + r = 36$

$p + q + r + t + 2t = 75$

$3t = 75 - 36 = 39$

$t = 13$

**I4.2** $k$ is a real number such that $k^4 + \frac{1}{k^4} = t + 1$, and $s = k^2 + \frac{1}{k^2}$. Find the value of $s$.

$k^4 + \frac{1}{k^4} = 14$

$k^4 + 2 + \frac{1}{k^4} = 16$

$(k^2 + \frac{1}{k^2})^2 = 16$

$\Rightarrow s = k^2 + \frac{1}{k^2} = 4$

**I4.3** $M$ and $N$ are the points (1, 2) and (11, 7) respectively. $P(a, b)$ is a point on $MN$ such that $MP : PN = 1 : s$. Find the value of $a$.

$MP : PN = 1 : 4$

$a = \frac{4+11}{1+4} = 3$

**I4.4** If the curve $y = ax^2 + 12x + c$ touches the $x$-axis, find the value of $c$.

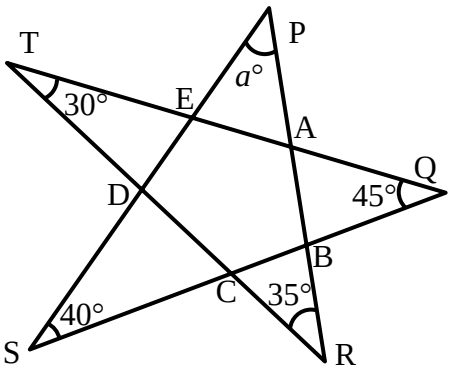
$y = 3x^2 + 12x + c$

$\Delta = 12^2 - 4(3)c = 0$

$\Rightarrow c = 12$

**Individual Event 5**

**I5.1** In the figure, find the value of $a$.




**Reference: 1997 FG1.1, 2005 FI2.3**

Label the vertices $A, B, C, D, E, P, Q, R, S, T$ as shown.

$\angle AEP = 40° + 45° = 85°$ (ext. $\angle$ of $\Delta SQE$)

$\angle EAP = 30° + 35° = 65°$ (ext. $\angle$ of $\Delta TRA$)

In $\Delta AEP$, $85° + 65° + a° = 180°$ ($\angle$s sum of $\Delta$)

$a = 30$

**I5.2** If $\sin(a° + 210°) = \cos b°$, and $90° < b < 180°$, find the value of $b$.

$\sin 240° = -\frac{\sqrt{3}}{2} = \cos b°$

$b = 150$

**I5.3** Each interior angle of an $n$-sided regular polygon is $b°$. Find the value of $n$.

Each exterior angle = 30° (adj. $\angle$s on st. line)

$\frac{360}{n} = 30$ (sum of exterior angles of polygon)

$\Rightarrow n = 12$

**I5.4** If the $n^{th}$ day of March in a year is Friday. The $k^{th}$ day of March in the same year is Wednesday, where $20 < k < 25$, find the value of $k$.

$12^{th}$ March is Friday

$17^{th}$ March is Wednesday

$24^{th}$ March is Wednesday

$\Rightarrow k = 24$

**Sample Group Event**

**SG.1** If $2at^2 + 12t + 9 = 0$ has equal roots, find the value of $a$.

$(12)^2 - 4(2a)(9) = 0$

$\Rightarrow a = 2$

**SG.2** If $ax + by = 1$ and $4x + 18y = 3$ are parallel, find the value of $b$.

**Reference: 1986 FI4.2, 1987 FSG.4**

$$-\frac{2}{b} = -\frac{4}{18}$$

$\Rightarrow b = 9$

**SG.3** The $b^{\text{th}}$ prime number is $p$. Find the value of $p$.

**Reference: 1985 FSG.2, 1990 FI5.4**

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ⋯

$p = 23$

**SG.4** If $k = \dfrac{4\sin\theta + 3\cos\theta}{2\sin\theta - \cos\theta}$ and $\tan\theta = 3$, find the value of $k$.

**Reference: 1986 FG10.3, 1987 FG8.1, 1989 FG10.3, 1990 FG7.2**

$$k = \frac{(4\sin\theta + 3\cos\theta) \div \cos\theta}{(2\sin\theta - \cos\theta) \div \cos\theta}$$

$$= \frac{4\tan\theta + 3}{2\tan\theta - 1}$$

$$= \frac{4(3) + 3}{2(3) - 1}$$

$= 3$

**Group Event 6**

**G6.1** An $n$-sided convex polygon has 20 diagonals. Find the value of $n$.

**Reference: 1984 FG10.3, 1985 FG8.3, 1988 FG6.2, 1991 FI2.3, 2001 FI4.2, 2005 FI1.4**

Number of diagonals $= C_2^n - n = \frac{n(n-1)}{2} - n = 20$

$n^2 - 3n - 40 = 0$

$(n - 8)(n + 5) = 0$

$\Rightarrow n = 8$

**G6.2** Two dice are thrown. The probability of getting a total of $n$ is $\frac{k}{36}$. Find the value of $k$.

Total = 8

Favourable outcomes = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

P(total = 8) $= \frac{5}{36}$

$k = 5$

**G6.3** A man drives at 25 km/h for 3 hours and then at 50 km/h for 2 hours.

His average speed for the whole journey is $u$ km/h. Find the value of $u$.

$u = \frac{25 \times 3 + 50 \times 2}{3 + 2} = 35$

**G6.4** If $a\Delta b = ab + 1$ and $(2\Delta a)\Delta 3 = 10$, find the value of $a$.

$2\Delta a = 2a + 1$

$(2\Delta a)\Delta 3 = (2a + 1)\Delta 3 = 3(2a + 1) + 1 = 10$

$6a + 4 = 10$

$a = 1$

**Group Event 7**

In the attached calculation, different letters represent different integers ranging from 1 to 9.

If the letters $O$ and $J$ represent 4 and 6 respectively, find the values of

**G7.1** $G$.

**G7.2** $D$.

**G7.3** $L$.

**G7.4** $E$.

Carry digit in the 100000 digit is 2

$G = 1$, $D = 8$

Carry digit in the hundreds digit is 3

$E = 5$

Carry digit in the tens digit is 4

$N = 7$, $L = 2$

$\therefore G = 1$, $D = 8$, $L = 2$, $E = 5$

| | | | | | | |
|---|---|---|---|---|---|---|
| | $G$ | $O$ | $L$ | $D$ | $E$ | $N$ |
| $\times$ | | | | | | $J$ |
| | $D$ | $E$ | $N$ | $G$ | $O$ | $L$ |

| | | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 4 | $L$ | 8 | $E$ | $N$ |
| $\times$ | | | | | | 6 |
| | 8 | $E$ | $N$ | 1 | 4 | $L$ |

| | | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 4 | 2 | 8 | 5 | 7 |
| $\times$ | | | | | | 6 |
| | 8 | 5 | 7 | 1 | 4 | 2 |

**Group Event 8**

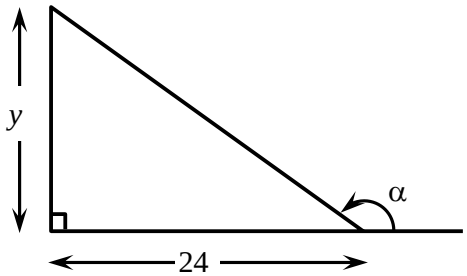
**G8.1** If $y$ is the greatest value of $\frac{14}{5+3\sin\theta}$, find the value of $y$.

$2 \le 5 + 3\sin\theta \le 8$

$\frac{14}{8} \le \frac{14}{5+3\sin\theta} \le \frac{14}{2}$

$\Rightarrow y = 7$

**G8.2** In the figure, $100\cos\alpha = k$. Find the value of $k$.




Hypotenuse = 25

$k = -100\cos(\alpha - 180°)$

$= -100 \cdot \frac{24}{25} = -96$

**G8.3** When $3x^2 + 4x + a$ is divided by $x + 2$, the remainder is 5. Find the value of $a$.

$3(-2)^2 + 4(-2) + a = 5$

$a = 1$

**G8.4** The solution for $3t^2 - 5t - 2 < 0$ is $-\frac{1}{3} < t < m$. Find the value of $m$.

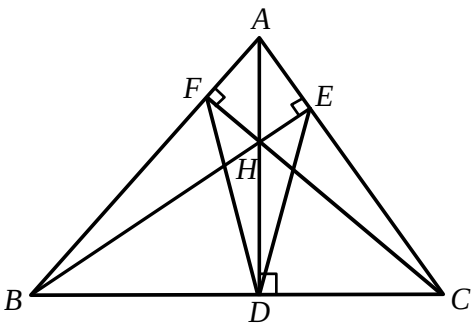
$(3t + 1)(t - 2) < 0$

$\Rightarrow -\frac{1}{3} < t < 2$

$\Rightarrow m = 2$

**Group Event 9**

**G9.1** In the figure, $\angle BAC = 70°$ and $\angle FDE = x°$. Find the value of $x$.




$\angle AFC = 90° = \angle ADC$ (given)

$ACDF$ is a cyclic quad (converse, $\angle$s in the same seg.)

$\angle BDF = \angle BAC = 70°$ (ext. $\angle$, cyclic quad.)

$\angle AEB = 90° = \angle ADB$ (given)

$ABDE$ is a cyclic quad (converse, $\angle$s in the same seg.)

$\angle CDE = \angle BAC = 70°$ (ext. $\angle$, cyclic quad.)

$\angle FDE = 180° - \angle BDF - \angle CDE$ (adj. $\angle$s on st. line)

$= 180° - 70° - 70° = 40°$

$\Rightarrow x = 40$

**G9.2** A cuboid is $y$ cm wide, 6 cm long and 5 cm high. Its surface area is 126 cm$^2$. Find the value of $y$.

$2(5y + 6y + 5\times 6) = 126$

$11y = 33$

$y = 3$

**G9.3** If $\log_9(\log_2 k) = \frac{1}{2}$, find the value of $k$.

$\log_2 k = \sqrt{9} = 3$

$k = 2^3 = 8$

**G9.4** If $a : b = 3 : 8$, $b : c = 5 : 6$ and $a : c = r : 16$, find the value of $r$.

$a : b : c = 15 : 40 : 48$

$a : c = 15 : 48 = 5 : 16$

$\Rightarrow r = 5$

**Group Event 10**

**G10.1** If $\dfrac{6\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = 3\sqrt{a}+6$, find the value of $a$.

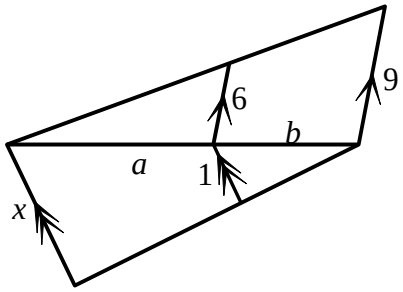
**Reference: 2014 FI4.1**

$\dfrac{6\sqrt{3}\left(3\sqrt{2}+2\sqrt{3}\right)}{18-12} = 3\sqrt{a}+6$

$3\sqrt{6}+6 = 3\sqrt{a}+6$

$a = 6$

**G10.2** In the figure, find the value of $x$.




**Reference: 1994 FI4.3**

By similar triangles

$6 : 9 = a : (a + b)$

$a = 2k, b = k$

$x : 1 = (a + b) : b = 3 : 1$

$x = 3$

**G10.3** If $k = \dfrac{6\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta}{\cos^2\theta + \sin\theta\cos\theta + \sin^2\theta}$ and $\tan\theta = 2$, find the value of $k$.

**Reference: 1986 FG10.3, 1987 FG8.1, 1989 FSG.4, 1990 FG7.2**

$k = \dfrac{\left(6\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta\right)\div\cos^2\theta}{\left(\cos^2\theta + \sin\theta\cos\theta + \sin^2\theta\right)\div\cos^2\theta}$

$= \dfrac{6 + 2\tan\theta + \tan^2\theta}{1 + \tan\theta + \tan^2\theta}$

$= \dfrac{6 + 2(2) + 2^2}{1 + 2 + 2^2}$

$= 2$

**G10.4** If $y = \dfrac{3\left(2^k\right) - 4\left(2^{k-2}\right)}{2^k - 2^{k-1}}$, find the value of $y$.

$y = \dfrac{3\left(2^k\right) - 4\left(2^{k-2}\right)}{2^k - 2^{k-1}}$

$= \dfrac{3-1}{1-\frac{1}{2}}$

$= 4$