Individual Events

SI	***a***	50	**I1**	***a***	15	**I2**	***a***	124	**I3**	***a***	7	**I4**	***a***	11	**I5**	***a***	1080
	b	10		***b***	3		***b***	50		***b***	125		***b***	5		***n***	21
	c	5		***c***	121		***n***	12		***c***	40		****c***	9 see the remark		***x***	25
	d	2		***d***	123		***d***	–10		***d***	50		***d***	5		***K***	6

Group Events

SG	***a***	64	**G6**	***M***	4	**G7**	***n***	5	**G8**	H_5	61	**G9**	**Area of Δ*BDF***	30	**G10**	***A***	3
	b	7		***N***	5		***c***	2		***a***	3		**Area of Δ*FDE***	75		***B***	1
	h	30		***z***	4		***x***	60		***t***	12		**Area of Δ*ABC***	28		***C***	5
	k	150		***r***	70		***y***	20		***m***	7		***x***	44		***D***	7

Sample Individual Event

SI.1 If $a = -1 + 2 - 3 + 4 - 5 + 6 - \ldots + 100$, find a.

Reference: 1998 FI2.4

$a = (-1 + 2 - 3 + 4) + (-5 + 6 - 7 + 8) + \ldots + (-97 + 98 - 99 + 100)$

$= 2 + 2 + \ldots + 2$ (25 terms) $= 50$

SI.2 The sum of the first b positive odd numbers is $2a$. Find b.

$1 + 3 + \ldots + (2b - 1) = 2a = 100$

$$\frac{b}{2}[2+2(b-1)]=100$$

$b^2 = 100$

$b = 10$

SI.3 A bag contains b white balls and 3 black balls. Two balls are drawn from the bag at random.

If the probability of getting 2 balls of different colours is $\frac{c}{13}$, find c.

The bag contains 10 white balls and 3 black balls.

P(2 different colours) $=2\times\frac{10}{13}\times\frac{3}{12}=\frac{5}{13}=\frac{c}{13}$

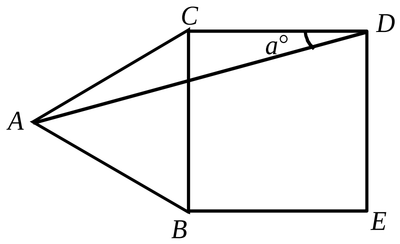
$c = 5$

SI.4 If the lines $cx + 10y = 4$ and $dx - y = 5$ are perpendicular to each other, find d.

$$-\frac{5}{10}\times\frac{d}{1}=-1$$

$\Rightarrow d = 2$

Individual Event 1



I1.1 In the figure, ABC is an equilateral triangle and $BCDE$ is a square. If $\angle ADC = a°$, find a. (**Reference 2014 FG3.3)**

$\angle ACD = (60 + 90)° = 150°$

$AC = CD$

$\angle CAD = a°$ (base, $\angle$s isos. Δ)

$a + a + 150 = 180$ ($\angle$s sum of Δ)

$a = 15$

I1.2 If $rb = 15$ and $br^4 = 125a$, where r is an integer, find b.

$br \cdot r^3 = 15r^3 = 125\times 15$

$\Rightarrow r^3 = 125$

$\Rightarrow r = 5$

$rb = 15$

$\Rightarrow b = 3$

I1.3 If the positive root of the equation $bx^2 - 252x - 13431 = 0$ is c, find c.

$3x^2 - 252x - 13431 = 0$

$\Rightarrow x^2 - 84x - 4477 = 0$, $4477 = 11\times 11\times 37$ and $-84 = -121 + 37$

$\Rightarrow (x - 121)(x + 37) = 0$

$\Rightarrow x = c = 121$

I1.4 Given $x \# y = \frac{y-1}{x} - x + y$. If $d = 10 \# c$, find d.

$d = 10 \# c$

$= \frac{121-1}{10} - 10 + 121$

$= 12 + 111 = 123$

Individual Event 2

I2.1 If $a^2 - 1 = 123\times125$ and $a > 0$, find a.

Reference: 1983 FI10.1, 1984 FSG.2

$a^2 - 1 = (124 - 1)\times(124 + 1)$

$= 124^2 - 1$

$a = 124$

I2.2 If the remainder of $x^3 - 16x^2 - 9x + a$ when divided by $x - 2$ is b, find b.

$b = 2^3 - 16(2)^2 - 9(2) + 124 = 50$

I2.3 If an n-sided polygon has $(b + 4)$ diagonals, find n.

Reference: 1984 FG10.3, 1985 FG8.3, 1988 FG6.2, 1989 FG6.1, 2001 FI4.2, 2005 FI1.4

$C_2^n - n = 50 + 4$

$n(n - 3) = 108$

$n^2 - 3n - 108 = 0$

$(n - 12)(n + 9) = 0$

$\Rightarrow n = 12$

I2.4 If the points $(3, n)$, $(5, 1)$ and $(7, d)$ are collinear, find d.

$\frac{12-1}{3-5} = \frac{d-1}{7-5}$

$d - 1 = -11$

$\Rightarrow d = -10$

Individual Event 3

I3.1 If the 6-digit number $168a26$ is divisible by 3, find the greatest possible value of a.

$1 + 6 + 8 + a + 2 + 6 = 3k$, where k is an integer.

The greatest possible value of $a = 7$

I3.2 A cube with edge a cm long is painted red on all faces. It is then cut into cubes with edge 1 cm long. If the number of cubes with all the faces not painted is b, find b.

Reference: 1994 HG2

The number of cubes with all the faces not painted is $b = (7 - 1 - 1)^3 = 125$

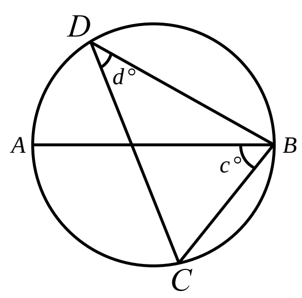
I3.3 If $(x - 85)(x - c) \equiv x^2 - bx + 85c$, find c.

$(x - 85)(x - c) \equiv x^2 - (85 + c)x + 85c$

$85 + c = b = 125$

$\Rightarrow c = 40$

I3.4 In the figure, AB is a diameter of the circle. Find d.



Label the vertices as shown.

$\angle CAB = d°$ ($\angle$ in the same segment)

$c + d = 90$ ($\angle$ in semi-circle)

$d = 50$

Individual Event 4

I4.1 Given $x - \frac{1}{x} = 3$. If $a = x^2 + \frac{1}{x^2}$, find a.

$a = (x - \frac{1}{x})^2 + 2$

$= 9 + 2 = 11$

I4.2 If $f(x) = \log_2 x$ and $f(a + 21) = b$, find b.

$b = f(11 + 21) = f(32)$

$= \log_2 32 = \log_2 2^5 = 5$

I4.3 If $\cos\theta = \frac{8b}{41}$, where θ is an acute angle, and $c = \frac{1}{\sin\theta} + \frac{1}{\tan\theta}$, find c.

$\cos\theta = \frac{40}{41}$

$\Rightarrow \sin\theta = \frac{9}{41}$, $\tan\theta = \frac{9}{40}$

$\Rightarrow c = \frac{41}{9} + \frac{40}{9} = 9$

Remark: Original question was: $\cdots\cdots$ where θ is **a positive** acute angle $\cdots\cdots$

Acute angle must be positive, the words "a positive" is replaced by "an".

I4.4 Two dice are tossed. If the probability of getting a sum of 7 or c is $\frac{d}{18}$, find d.

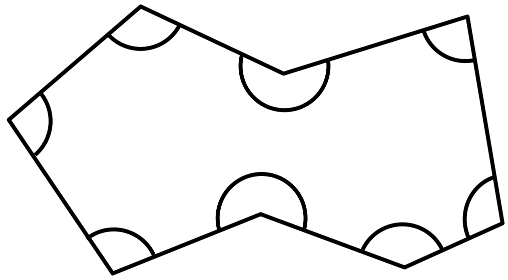
P(sum = 7 or 9) = P(7) + P(9)

$= \frac{6}{36} + \frac{4}{36} = \frac{5}{18}$

$\Rightarrow d = 5$

Individual Event 5

I5.1 In Figure 1, if the sum of the interior angles is $a°$, find a.



$a = 180\times(8-2)$ (∠s sum of polygon)

$a = 1080$

I5.2 If the n^{th} term of the arithmetic progression 80, 130, 180, 230, 280, … is a, find n.

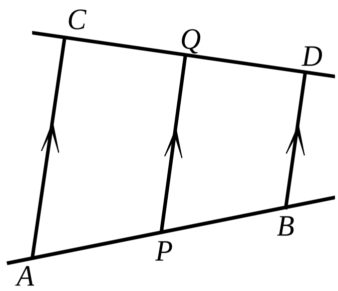
First term = 80, common difference = 50

$80 + (n-1)\cdot 50 = 1080$

$\Rightarrow n = 21$

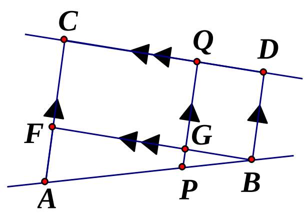
I5.3 In Figure 2, $AP : PB = 2 : 1$.

If $AC = 33$ cm, $BD = n$ cm, $PQ = x$ cm, find x.



Reference: 1986 FI3.3

From B, draw a line segment FGB // CQD, cutting AC, PQ at F and G respectively.



$CDBF$, $BDQG$ are parallelograms (2 pairs of // lines)

$CF = QG = DB = 21$ cm (opp. sides //-gram)

$AF = (33-21)\text{cm} = 12$ cm

$\Delta BPG \sim \Delta BAF$ (equiangular)

$$\frac{PG}{AF} = \frac{PB}{AP+PB} \quad \text{(ratio of sides, }\sim\Delta\text{s)}$$

$$\frac{PG}{12\text{ cm}} = \frac{1}{3}$$

$\Rightarrow PG = 4$ cm

$PQ = PG + GQ = (4 + 21)$ cm $= 25$ cm

$x = 25$

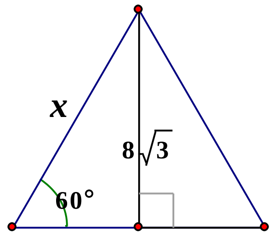
I5.4 If $K = \dfrac{\sin 65^\circ \tan^2 60^\circ}{\tan 30^\circ \cos 30^\circ \cos x^\circ}$, find K.

$$K = \frac{\sin 65^\circ \tan^2 60^\circ}{\tan 30^\circ \cos 30^\circ \cos 25^\circ}$$

$$= \frac{\sin 65^\circ \cdot \left(\sqrt{3}\right)^2}{\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} \cdot \sin 65^\circ} = 6$$

Sample Group Event

SG.1 The height of an equilateral triangle is $8\sqrt{3}$ cm and the area of the triangle is $a\sqrt{3}$ cm^2. Find a.



Let the length of a side be x cm.

In the figure, $x \sin 60° = 8\sqrt{3}$

$\Rightarrow x = 16$

Area $= \frac{1}{2} \cdot x^2 \sin 60°$

$= \frac{1}{2} \cdot 16^2 \cdot \frac{\sqrt{3}}{2} = a\sqrt{3}$

$\Rightarrow a = 64$

SG.2 Given that $\sum_{x=1}^{n} \frac{1}{x} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$, and $\sum_{x=4}^{10} \frac{1}{x-2} - \sum_{x=4}^{10} \frac{1}{x-1} = \frac{b}{18}$. Find b.

Reference: 1983 FG7.4

$\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}\right)-\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}\right)$

$= \frac{1}{2} - \frac{1}{9} = \frac{b}{18}$

$\Rightarrow b = 7$

SG.3-SG.4 A boy tries to find the area of a parallelogram by multiplying together the lengths of two adjacent sides. His answer is double the correct answer. If the acute angle and the obtuse angle of the figure are $h°$ and $k°$ respectively,

Reference: 1989 HI7

SG.3 find h.

Let the two adjacent sides be x and y.

$xy = 2 \cdot xy \sin h°$

$\Rightarrow \sin h° = \frac{1}{2}$

$\Rightarrow h = 30$

SG.4 find k.

$k = 180 - 30 = 150$ (int. ∠s, // lines)

Group Event 6

G6.1-6.2 A 2-digit number x has M as the units digit and N as the tens digit. Another 2-digit number y has N as the units digit and M as the tens digit. If $x > y$ and their sum is equal to eleven times their differences,

Reference: 1983 FG10.4

G6.1 find M. **G6.2** find N.

$x = 10N + M$, $y = 10M + N$

$x > y \Rightarrow N > M > 0$

$x + y = 11(x - y)$

$10N + M + 10M + N = 11(10N + M - 10M - N)$

$M + N = 9N - 9M$

$10M = 8N$

$5M = 4N$

M is a multiple of 4 and N is a multiple of 5.

$N = 5$, $M = 4$

G6.3 The sum of two numbers is 20 and their product is 5.

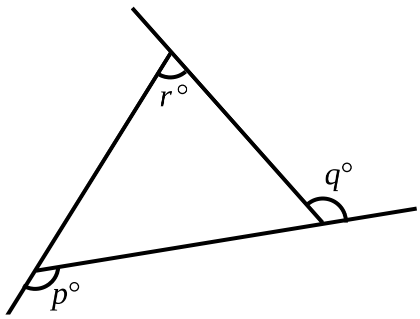
If the sum of their reciprocals is z, find z.

Let the 2 numbers be x and y.

$x + y = 20$ and $xy = 5$

$$z = \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = 4$$

G6.4 In the figure, the average of p and q is $121 + z$. Find r.



Reference: 1983 FG6.2

The exterior angle of $r°$ is $180° - r°$ (adj. ∠s on st. line)

$p + q + (180 - r) = 360$ (sum of ext. ∠s of polygon)

$p + q - r = 180$ …… (1)

$\frac{p+q}{2} = 121 + z = 125$

$\Rightarrow p + q = 250$ …… (2)

Sub. (2) into (1): $250 - r = 180$

$\Rightarrow r = 70$

Group Event 7

G7.1 5 printing machines can print 5 books in 5 days.

If n printing machines are required in order to have 100 books printed in 100 days, find n.

100 printing machines can print 100 books in 5 days.

5 printing machines can print 100 books in 100 days

$\Rightarrow n = 5$

G7.2 If the equation $x^2 + 2x + c = 0$ has no real root and c is an integer less than 3, find c.

$\Delta = 2^2 - 4c < 0$

$\Rightarrow c > 1$ and c is an integer less than 3

$\Rightarrow c = 2$

G7.3-G7.4 Chicken eggs cost \$0.50 each, duck eggs cost \$0.60 each and goose eggs cost \$0.90 each. A man sold x chicken eggs, y duck eggs, z goose eggs and received \$60. If x, y, z are all positive numbers with $x + y + z = 100$ and two of the values x, y, z are equal,

G7.3 find x. **G7.4** find y.

$0.5x + 0.6y + 0.9z = 60$

$\Rightarrow 5x + 6y + 9z = 600 \cdots\cdots (1)$

$x + y + z = 100 \cdots\cdots (2)$

If $x = z$, then $14x + 6y = 600$

$\Rightarrow 7x + 3y = 300 \cdots\cdots (3)$ and $2x + y = 100 \cdots\cdots (4)$

(3) – 3(4): $x = 0$ (rejected)

If $x = y$, then $11x + 9z = 600 \cdots\cdots (5)$ and $2x + z = 100 \cdots\cdots (6)$

9(6) – (5): $7x = 300$, x is not an integer, rejected.

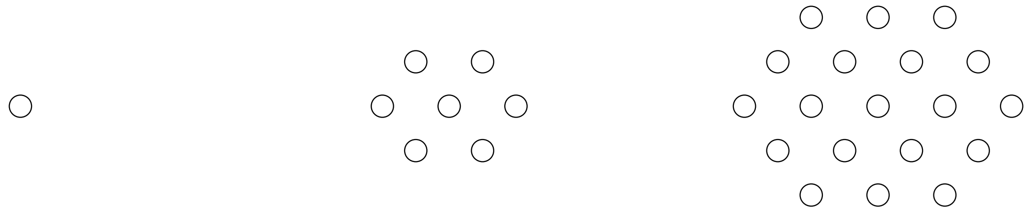
(1) – 5(2): $y + 4z = 100 \cdots\cdots (7)$

If $y = z$, then $y = z = 20$, $x = 60$

Group Event 8

Reference: 1992 FG9.3-4

G8.1-G8.2 Consider the following hexagonal numbers :



$H_1 = 1$ $H_2 = 7$ $H_3 = 19$

G8.1 Find H_5.

$H_2 - H_1 = 6\times1$, $H_3 - H_2 = 12 = 6\times2$

$H_4 - H_3 = 18 = 6\times3$

$\Rightarrow H_4 = 19 + 18 = 37$

$H_5 - H_4 = 6\times4 = 24$

$\Rightarrow H_5 = 24 + 37 = 61$

G8.2 If $H_n = an^2 + bn + c$, where n is any positive integer, find a.

$H_1 = a + b + c = 1 \cdots\cdots (1)$

$H_2 = 4a + 2b + c = 7 \cdots\cdots (2)$

$H_3 = 9a + 3b + c = 19 \cdots\cdots (3)$

(2) – (1): $3a + b = 6 \cdots\cdots (4)$

(3) – (2): $5a + b = 12 \cdots\cdots (5)$

(5) – (4): $2a = 6$

$\Rightarrow a = 3$

G8.3 If $p : q = 2 : 3$, $q : r = 4 : 5$ and $p : q : r = 8 : t : 15$, find t.

$p : q : r = 8 : 12 : 15$

$\Rightarrow t = 12$

G8.4 If $\frac{1}{x} : \frac{1}{y} = 4 : 3$ and $\frac{1}{x+y} : \frac{1}{x} = 3 : m$, find m.

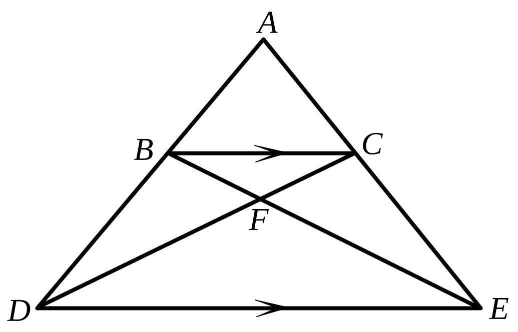
$x : y = \frac{1}{4} : \frac{1}{3} = 3 : 4$

$\frac{1}{x+y} : \frac{1}{x} = \frac{1}{3+4} : \frac{1}{3} = 3 : 7$

$m = 7$

Group Event 9

G9.1-G9.3



In the figure, *BC* is parallel to *DE*.

If $AB : BC : BF : CF : FE = 5 : 4 : 2 : 3 : 5$

and the area of ΔBCF is 12, find

G9.1 the area of ΔBDF,

G9.2 the area of ΔFDE,

G9.3 the area of ΔABC.

G9.1 $\Delta BCF \sim \Delta EDF$ (equiangular)

$DF : EF : DE = CE : FB : BC$ (ratio of sides, ~Δs)

$DF = 3\times\frac{5}{2} = 7.5$, $DE = 4\times\frac{5}{2} = 10$

The area of $\Delta BDF = 12\times\frac{7.5}{3} = 30$

G9.2 The area of $\Delta FDE = 30\times\frac{5}{2} = 75$

G9.3 The area of $\Delta CEF = 12\times\frac{5}{2} = 30$

The area of $BCED = 12 + 30 + 30 + 75 = 147$

$\Delta ABC \sim \Delta ADE$ (equiangular)

Area of ΔABC : area of $\Delta ADE = BC^2 : DE^2 = 4^2 : 10^2 = 4 : 25$

Let the area of ΔABC be y

$y : (y + 147) = 4 : 25$

$4y + 588 = 25y$

$21y = 588$

$y =$ Area of $\Delta ABC = 28$

G9.4 If the volume of a sphere is increased by 72.8%, then the surface area of the sphere is increased by x%. Find x.

Let the original radius of the sphere be r and the new radius be R

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3\cdot(1+72.8\%)$$

$$\left(\frac{R}{r}\right)^3 = 1.728 = 1.2^3$$

$\Rightarrow R = 1.2r$

$\Rightarrow x = 20$

Group Event 10

In the attached division

G10.1 find A,

G10.2 find B,

G10.3 find C,

G10.4 find D.

$$
\begin{array}{r|ccccc}
 & & & 1 & D & E \\ \hline
2\ 1\ 5 & A & 7 & B & 9 & C \\
 & F & G & H & & \\ \hline
 & J & 5 & K & 9 & \\
 & L & 5 & M & 5 & \\ \hline
 & & & N & 4 & P \\
 & & & Q & R & S \\ \hline
\end{array}
$$

$\overline{FGH} = 215$

$D \geq 5$

$D = 5, 7, 9$

$215\times5 = 1055 \neq \overline{L5M5}$ (rejected)

$215\times7 = 1505 = \overline{L5M5}$ (accepted)

$215\times9 = 1935 \neq \overline{L5M5}$ (rejected)

$\therefore\ D = 7, L = 1, M = 0$

$$
\begin{array}{r|ccccc}
 & & & 1 & D & E \\ \hline
2\ 1\ 5 & A & 7 & B & 9 & C \\
 & 2 & 1 & 5 & & \\ \hline
 & J & 5 & K & 9 & \\
 & L & 5 & M & 5 & \\ \hline
 & & & N & 4 & P \\
 & & & Q & R & S \\ \hline
\end{array}
$$

$J = 1, A = 3$

$E = 2, 3$ or 4

$\overline{N4P} = \overline{QRS}$

$215\times2 = 430 \neq \overline{N4P}$ (rejected)

$215\times3 = 645 = \overline{N4P}$ (accepted)

$215\times4 = 860 \neq \overline{N4P}$ (rejected)

$\therefore\ E = 3$

$$
\begin{array}{r|ccccc}
 & & & 1 & 7 & E \\ \hline
2\ 1\ 5 & A & 7 & B & 9 & C \\
 & 2 & 1 & 5 & & \\ \hline
 & J & 5 & K & 9 & \\
 & 1 & 5 & 0 & 5 & \\ \hline
 & & & N & 4 & P \\
 & & & Q & R & S \\ \hline
\end{array}
$$

$215\times173 = 37195$

$\therefore\ A = 3, B = 1, C = 5, D = 7$

$$
\begin{array}{r|ccccc}
 & & & 1 & 7 & 3 \\ \hline
2\ 1\ 5 & 3 & 7 & 1 & 9 & 5 \\
 & 2 & 1 & 5 & & \\ \hline
 & 1 & 5 & 6 & 9 & \\
 & 1 & 5 & 0 & 5 & \\ \hline
 & & & 6 & 4 & 5 \\
 & & & 6 & 4 & 5 \\ \hline
\end{array}
$$