Individual Events

SI	A	20	I1	n	10	I2	a	48	I3	a	2	I4	A	40	I5	a	45
	B	4		a	25		b	144		b	–3		B	6		b	15
	C	5		z	205		c	4		c	12		C	198		c	12
	D	$\frac{5}{2}$		S	1		d	572		d	140		D	7		d	2

Group Events

SG		2550	G6	a	1	G7	a	–8	G8	A	2	G9	x	6	G10	c	3
		2452		b	52		b	10		b	171		y	6		a	–2
	P	2501		c	13		**area**	116		c	3		T_{10}	200		b	5
	Q	10001		d	3		**tan θ**	2		d	27		n	19		d	5

Sample Individual Event

SI.1 Given $A = (b^m)^n + b^{m+n}$. Find the value of A when $b = 4$, $m = n = 1$.

$A = (4^1)^1 + 4^{1+1} = 4 + 16 = 20$

SI.2 If $2^A = B^{10}$ and $B > 0$, find the value of B .

$2^{20} = 4^{10}$

$\Rightarrow B = 4$

SI.3 Solve for C in the following equation: $\sqrt{\frac{20B+45}{C}} = C$.

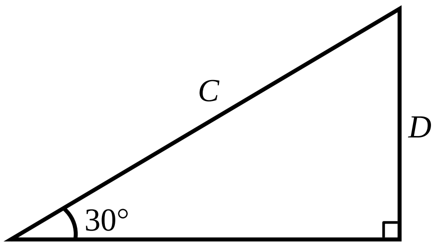
$\sqrt{\frac{20\times 4+45}{C}} = C$

$125 = C^3$

$\Rightarrow C = 5$

SI.4 Find the value of D in the figure.

$D = C \sin 30° = \frac{5}{2}$



Individual Event 1

I1.1 If the sum of the interior angles of an n-sided polygon is 1440°, find the value of n .

$180°\times(n-2) = 1440°$

$\Rightarrow n = 10$

I1.2 If $x^2 - nx + a = 0$ has 2 equal roots, find the value of a .

$(-10)^2 - 4a = 0$

$\Rightarrow a = 25$

I1.3 In the figure, if $z = p + q$, find the value of z .

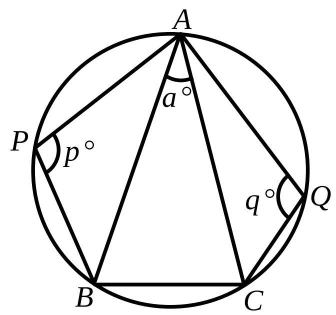
Reference: 1989 HI19

$\angle ACB = 180° - p°$ (opp. $\angle$s cyclic quad.)

$\angle ABC = 180° - q°$ (opp. $\angle$s cyclic quad.)

$180 - p + 180 - q + a = 180$ ($\angle$s sum of Δ)

$z = p + q = 180 + a = 205$



I1.4 If $S = 1 + 2 - 3 - 4 + 5 + 6 - 7 - 8 + \cdots + z$, find the value of S .

Reference: 1985 FG7.4, 1988 FG6.4, 1990 FG10.1, 1991 FSI.1

$S = 1 + (2 - 3 - 4 + 5) + (6 - 7 - 8 + 9) + \cdots + (202 - 203 - 204 + 205) = 1$

Individual Event 2

I2.1 If $ar = 24$ and $ar^4 = 3$, find the value of a.

$r^3 = \frac{ar^4}{ar} = \frac{3}{24} = \frac{1}{8}$

$\Rightarrow r = \frac{1}{2}$

$ar = 24$

$\Rightarrow \frac{1}{2}a = 24$

$\Rightarrow a = 48$

I2.2 If $\left(x+\frac{a}{4}\right)^2 = x^2 + \frac{a}{2}\cdot x + b$, find the value of b.

$(x + 12)^2 = x^2 + 24x + 144$

$\Rightarrow b = 144$

I2.3 If $c = \log_2 \frac{b}{9}$, find the value of c.

$c = \log_2 \frac{144}{9}$

$= \log_2 16$

$= 4$

I2.4 If $d = 12^c - 142^2$, find the value of d.

$d = 12^4 - 142^2$

$= 144^2 - 142^2$

$= (144 + 142)(144 - 142)$

$= 2(286) = 572$

Individual Event 3

I3.1 If $a=\dfrac{\sin 15^\circ}{\cos 75^\circ}+\dfrac{1}{\sin^2 75^\circ}-\tan^2 15^\circ$, find the value of a.

$a=\dfrac{\sin 15^\circ}{\sin 15^\circ}+\sec^2 15^\circ-\tan^2 15^\circ$

$=1+1=2$

I3.2 If the lines $ax+2y+1=0$ and $3x+by+5=0$ are perpendicular to each other, find the value of b.

$-\dfrac{a}{2}\times\left(-\dfrac{3}{b}\right)=-1$

$\Rightarrow b=-3$

I3.3 The three points $(2, b)$, $(4, -b)$ and $(5, \frac{c}{2})$ are collinear. Find the value of c.

The three points are $(2, -3)$, $(4, 3)$ and $(5, \frac{c}{2})$, so their slopes are equal.

$\dfrac{3-(-3)}{4-2}=\dfrac{\frac{c}{2}-3}{5-4}$

$\Rightarrow \dfrac{c}{2}-3=3$

$\Rightarrow c=12$

I3.4 If $\dfrac{1}{x}:\dfrac{1}{y}:\dfrac{1}{z}=3:4:5$ and $\dfrac{1}{x+y}:\dfrac{1}{y+z}=9c:d$, find the value of d.

$x:y:z=\dfrac{1}{3}:\dfrac{1}{4}:\dfrac{1}{5}$

$=\dfrac{20}{60}:\dfrac{15}{60}:\dfrac{12}{60}$

$=20:15:12$

$x=20k, y=15k, z=12k$

$\dfrac{1}{x+y}:\dfrac{1}{y+z}=\dfrac{1}{20k+15k}:\dfrac{1}{15k+12k}$

$=27:35$

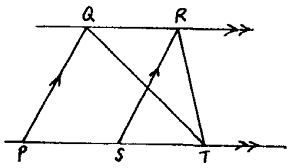
$=108:140=9c:d$

$\Rightarrow d=140$

Individual Event 4

I4.1 In the figure, the area of $PQRS$ is 80 cm^2.

If the area of ΔQRT is A cm^2, find the value of A.



ΔQRT has the same base and same height as the parallelogram $PQRS$.

$$A = \frac{1}{2} \cdot 80 = 40$$

I4.2 If $B = \log_2\left(\frac{8A}{5}\right)$, find the value of B.

$$B = \log_2\left(\frac{8 \cdot 40}{5}\right)$$

$= \log_2 64$

$= \log_2 2^6$

$= 6$

I4.3 Given $x + \frac{1}{x} = B$. If $C = x^3 + \frac{1}{x^3}$, find the value of C.

$$x + \frac{1}{x} = 6$$

$$x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2$$

$= 6^2 - 2 = 34$

$$C = x^3 + \frac{1}{x^3}$$

$$= \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)$$

$= 6(34 - 1) = 198$

I4.4 Let $(p, q) = qD + p$. If $(C, 2) = 212$, find the value of D.

$2D + C = 212$

$\Rightarrow 2D = 212 - 198 = 14$

$\Rightarrow D = 7$

Individual Event 5

I5.1 Let p, q be the roots of the quadratic equation $x^2 - 3x - 2 = 0$ and $a = p^3 + q^3$. Find the value of a.

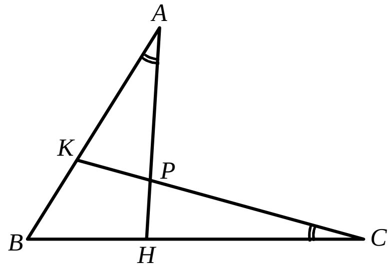
$p + q = 3$, $pq = -2$

$a = (p + q)(p^2 - pq + q^2)$

$= 3[(p + q)^2 - 3pq]$

$= 3[3^2 - 3(-2)] = 45$

I5.2 If $AH = a$, $CK = 36$, $BK = 12$ and $BH = b$, find the value of b.

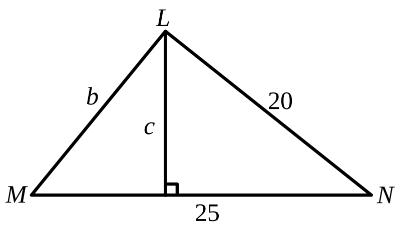


$\Delta ABH \sim \Delta CBK$ (equiangular)

$\frac{b}{12} = \frac{45}{36}$ (ratio of sides, $\sim\Delta$s)

$b = 15$

I5.3 Find the value of c.



Reference: 1985 FG6.4

$15^2 + 20^2 = 25^2$

$\Rightarrow ML \perp LN$ (converse, Pythagoras' theorem)

Area of $\Delta MNL = \frac{1}{2}\cdot 15\cdot 20 = \frac{1}{2}\cdot 25c$

$c = 12$

I5.4 Let $\sqrt{2x+23}+\sqrt{2x-1}=c$ and $d=\sqrt{2x+23}-\sqrt{2x-1}$. Find the value of d.

Reference: 2014 HG1

$cd = \left(\sqrt{2x+23}+\sqrt{2x-1}\right)\left(\sqrt{2x+23}-\sqrt{2x-1}\right)$

$12d = (2x + 23) - (2x - 1) = 24$

$\Rightarrow d = 2$

Sample Group Event Reference HKCEE Mathematics 1990 Paper 1 Q14

Consider the following groups of numbers:

(2)

(4, 6)

(8, 10, 12)

(14, 16, 18, 20)

(22, 24, 26, 28, 30)

……………………………

SG.1 Find the last number of the 50th group.

$2 = 2\times 1$

$6 = 2(1 + 2)$

$12 = 2(1 + 2 + 3)$

$20 = 2(1 + 2 + 3 + 4)$

$30 = 2(1 + 2 + 3 + 4 + 5)$

The last number of the 50th group

$= 2(1 + 2 + \cdots + 50)$

$= 2\cdot\frac{1}{2}\cdot 50\cdot(1+50) = 2550$

SG.2 Find the first number of the 50th group.

There are 50 numbers in the 50th group.

The first number of the 50th group $= 2550 - 2(50 - 1) = 2452$

SG.3 Find the value of P if the sum of the numbers in the 50th group is $50P$.

$2452 + 2454 + \cdots + 2550 = 50P$

$\frac{1}{2}\cdot 50\cdot(2452+2550) = 50P$

$P = 2501$

SG.4 Find the value of Q if the sum of the numbers in the 100th group is $100Q$.

The last number in the 100th group $= 2(1 + 2 + \cdots + 100) = 2\cdot\frac{1}{2}\cdot 100\cdot(1+100) = 10100$

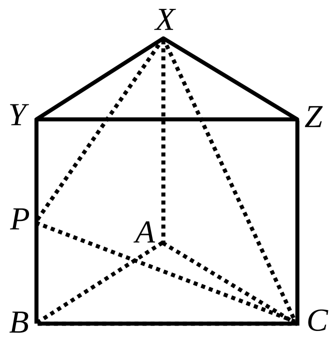
The first number of the 100th group $= 10100 - 2(100 - 1) = 9902$

$9902 + 9904 + \cdots + 10100 = 100P$

$\frac{1}{2}\cdot 100\cdot(9902+10100) = 100P$

$P = 10001$

Group Event 6



As shown in the figure, ΔABC and ΔXYZ are equilateral triangles and are ends of a right prism. P is the mid-point of BY and $BP = 3$ cm, $XY = 4$ cm.

G6.1 If $a = \dfrac{CP}{PX}$, find the value of a.

$CP = \sqrt{3^2 + 4^2}$ cm = 5 cm = PX (Pythagoras' theorem)

$a = 1$

G6.2 If $CX = \sqrt{b}$ cm, find the value of b.

$CX = \sqrt{6^2 + 4^2}$ cm = $\sqrt{52}$ cm (Pythagoras' theorem)

$b = 52$

G6.3 If $\cos\angle PCX = \dfrac{\sqrt{c}}{5}$, find the value of c.

$\cos\angle PCX = \dfrac{\sqrt{52} \div 2}{5} = \dfrac{\sqrt{13}}{5}$

$\Rightarrow c = 13$

G6.4 If $\sin\angle PCX = \dfrac{2\sqrt{d}}{5}$, find the value of d.

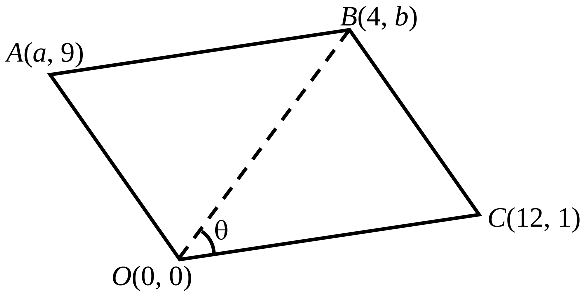
$\sin^2\angle PCX = 1 - \cos^2\angle PCX = \dfrac{12}{25}$

$\sin\angle PCX = \dfrac{2\sqrt{3}}{5}$

$\Rightarrow d = 3$

Group Event 7

In the figure, $OABC$ is a parallelogram.



G7.1 Find the value of a.

$a - 0 = 4 - 12$

$\Rightarrow a = -8$

G7.2 Find the value of b.

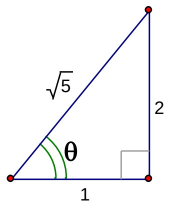
$b - 1 = 9 - 0$

$\Rightarrow b = 10$

G7.3 Find the area of $OABC$.

$$\text{Area} = 2 \cdot \frac{1}{2}\begin{vmatrix} 0 & 0 \\ 12 & 1 \\ 4 & 10 \\ 0 & 0 \end{vmatrix} = 116$$

G7.4 Find the value of $\tan\theta$.



$OC = \sqrt{145}$

$OB = \sqrt{116}$

$BC = \sqrt{(12-4)^2 + (1-10)^2} = \sqrt{145}$

$$\cos\theta = \frac{\sqrt{145}^2 + \sqrt{116}^2 - \sqrt{145}^2}{2(\sqrt{145})(\sqrt{116})} = \frac{1}{\sqrt{5}}$$

$\tan\theta = 2$

Method 2

$$m_{OC} = \frac{1-0}{12-0} = \frac{1}{12}$$

$$m_{OB} = \frac{10-0}{4-0} = \frac{5}{2}$$

$$\tan\theta = \frac{\frac{5}{2} - \frac{1}{12}}{1 + \frac{5}{2}\cdot\frac{1}{12}} = 2$$

Group Event 8

G8.1 The area of an equilateral triangle of side A cm is $\sqrt{3}$ cm^2. Find the value of A.

$\frac{1}{2}\cdot A^2\sin 60^\circ=\sqrt{3}$

$\Rightarrow A = 2$

G8.2 If $19\times 243^{\frac{A}{5}}=b$, find the value of b.

$b=19\times\left(3^5\right)^{\frac{2}{5}}=171$

G8.3 The roots of the equation $x^3 - 173x^2 + 339x + 513 = 0$ are -1, b and c. Find the value of c.

$-1 + 171 + c =$ sum of roots $= 173$

$\Rightarrow c = 3$

G8.4 The base of a triangular pyramid is an equilateral triangle of side $2c$ cm.

If the height of the pyramid is $\sqrt{27}$ cm, and its volume is d cm^3, find the value of d.

$d=\frac{1}{3}\cdot\frac{1}{2}\cdot\left(6^2\cdot\sin 60^\circ\right)\cdot\sqrt{27}=27$

Group Event 9

If the area of a regular hexagon $ABCDEF$ is $54\sqrt{3}$ cm^2 and $AB = x$ cm, $AC = y\sqrt{3}$ cm,

G9.1 find the value of x .

The hexagon can be cut into 6 identical equilateral triangles

$6\cdot\frac{1}{2}\cdot\left(x^2\cdot\sin 60^\circ\right)=54\sqrt{3}$

$\Rightarrow x = 6$

G9.2 find the value of y .

$\angle ABC = 120°$

$AC^2 = (x^2 + x^2 - 2x^2 \cos 120°)$ cm^2

$= [6^2 + 6^2 - 2(6)^2\cdot\left(-\frac{1}{2}\right)]$ cm^2

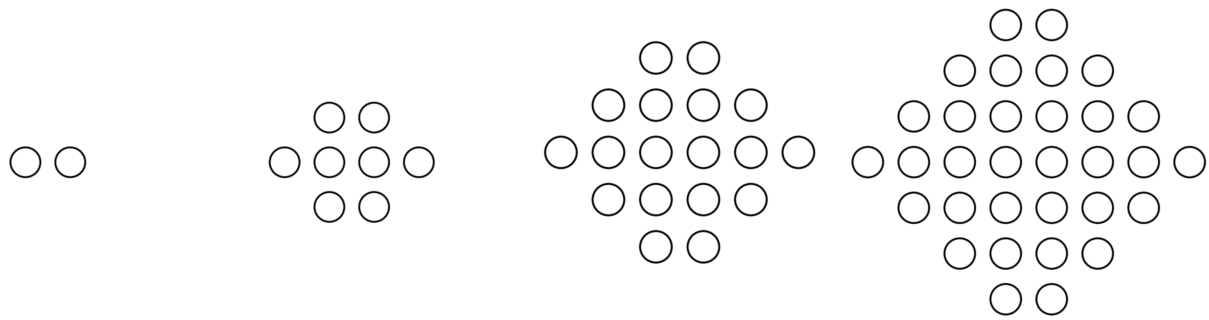
$= 3\times6^2$ cm^2

$y\sqrt{3}=6\sqrt{3}$

$\Rightarrow y = 6$

G9.3 - G9.4 (Reference: 1991 FG8.1-2)

Consider the following number pattern:



$T_1 = 2$ $T_2 = 8$ $T_3 = 18$ $T_4 = 32$

G9.3 Find the value of T_{10} .

$8 - 2 = 6$, $18 - 8 = 10$, $32 - 18 = 14$

$\Rightarrow T_1 = 2$, $T_2 = 2 + 6$, $T_3 = 2 + 6 + 10$, $T_4 = 2 + 6 + 10 +14$

$T_{10}=\frac{10}{2}\cdot[2(2)+(10-1)\cdot 4]= 200$

G9.4 If $T_n = 722$, find the value of n .

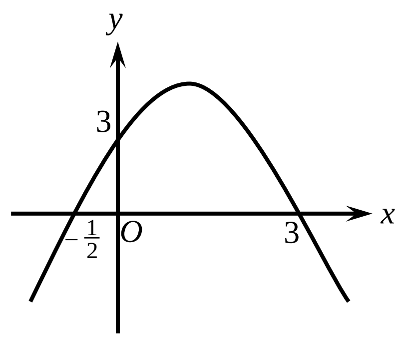
$\frac{n}{2}\cdot[2(2)+(n-1)\cdot 4]=722$

$n^2 = 361$

$n = 19$

Group Event 10

The following shows the graph of $y = ax^2 + bx + c$.



G10.1 Find the value of c .

$x = 0, y = c = 3$

G10.2 Find the value of a .

$y = a(x + \frac{1}{2})(x - 3)$

Sub. $x = 0, y = 3$

$\Rightarrow -\frac{3}{2}a = 3$

$a = -2$

G10.3 Find the value of b .

$3 - \frac{1}{2}$ = sum of roots $= -\frac{b}{(-2)}$

$b = 5$

G10.4 If $y = x + d$ is tangent to $y = ax^2 + bx + c$, find the value of d .

Sub. $y = x + d$ into $y = ax^2 + bx + c$

$-2x^2 + 5x + 3 = x + d$

$2x^2 - 4x + d - 3 = 0$

$\Delta = (-4)^2 - 4(2)(d - 3) = 0$

$4 - 2d + 6 = 0$

$d = 5$