Individual Events

SI	***a***	$\frac{2}{3}$	**I1**	***a***	*1 see the remark	**I2**	***a***	38	**I3**	***a***	10	**I4**	***p***	15	**I5**	***a***	4
	b	0		***b***	2		***b***	104		***b***	27		***q***	4		***b***	5
	c	3		***c***	4		***c***	100		***c***	*23 see the remark		***r***	57		***c***	24
	d	–6		***d***	24		***d***	–50		***d***	26		***s***	3		***d***	57

Group Events

SG	***a***	10	**G1**	***p***	4	**G2**	***a***	1110	**G3**	***a***	90	**G4**	***a***	0.13717421	**G5**	***a***	4290	**GS**	***s***	6
	b	73		***q***	3		***b***	1		***b***	1		***b***	90		***b***	18		***b***	10
	c	55		***r***	2		***c***	0		***c***	0		***c***	$\frac{665}{729}$		***c***	67		***c***	81
	d	16		***a***	9		***d***	6		***d***	1		***d***	50		***d***	30		***d***	50

Sample Individual Event (1997 Final Individual Event 1)

SI.1 Given that $\frac{3}{a}+\frac{1}{u}=\frac{7}{2}$ and $\frac{2}{a}-\frac{3}{u}=6$ are simultaneous equations in a and u. Solve for a.

3(1) + (2): $\frac{11}{a}=\frac{33}{2}$

$a=\frac{2}{3}$

SI.2 Three solutions of the equation $px + qy + bz = 1$ are $(0, 3a, 1)$, $(9a, –1, 2)$ and $(0, 3a, 0)$. Find the value of the coefficient b.

$$\begin{cases} 3aq+b=1 \\ 9ap-q+2b=1 \\ 3aq=1 \end{cases}$$

Sub. (3) into (1): $1 + b = 1$

$\Rightarrow b = 0$

SI.3 Find c so that the graph of $y = mx + c$ passes through the two points $(b + 4, 5)$ and $(–2, 2)$.

The 2 points are: (4, 5) and (–2, 2). The slope is $\frac{5-2}{4-(-2)}=\frac{1}{2}$.

The line $y=\frac{1}{2}x+c$ passes through (–2, 2): $2 = –1 + c$

$\Rightarrow c = 3$

SI.4 The solution of the inequality $x^2 + 5x – 2c \le 0$ is $d \le x \le 1$. Find d.

$x^2 + 5x – 6 \le 0$

$\Rightarrow (x + 6)(x – 1) \le 0$

$–6 \le x \le 1$

$d = –6$

Individual Event 1

I1.1 If a is the maximum value of $\frac{1}{2}\sin^2 3\theta - \frac{1}{2}\cos 2\theta$, find the value of a.

$-1 \le \sin 3\theta \le 1$ and $-1 \le \cos 2\theta \le 1$

$\frac{1}{2}\sin^2 3\theta \le \frac{1}{2}$ and $-\frac{1}{2}\cos 2\theta \le \frac{1}{2}$

$\frac{1}{2}\sin^2 3\theta - \frac{1}{2}\cos 2\theta \le \frac{1}{2} + \frac{1}{2} = 1 = a$,

Maximum occur when $\sin^2 3\theta = 1$ and $-\cos 2\theta = 1$

i.e. $3\theta = 90° + 180°n$ and $2\theta = 360°m + 180°$, where m, n are integers.

$\theta = 30° + 60°n = 180°m + 90° \Rightarrow 60°n = 180°m + 60° \Rightarrow n = 3m + 1$; let $m = 1$, $n = 4, \theta = 270°$

Remark: the original question is

If a is the maximum value of $\frac{1}{2}\sin^2 \theta + \frac{1}{2}\cos 3\theta$, find the value of a.

Maximum occur when $\sin^2 \theta = 1$ and $\cos 3\theta = 1$

i.e. $\theta = 90° + 180°n$ and $3\theta = 360°m$, where m, n are integers.

$\theta = 90° + 180°n = 120°m \Rightarrow 3 + 6n = 4m$, LHS is odd and RHS is even, contradiction.

The question was wrong because we cannot find any θ to make the expression a maximum.

I1.2 If $\begin{cases} x + y = 2 \\ xy - z^2 = a \\ b = x + y + z \end{cases}$, find the value of b.

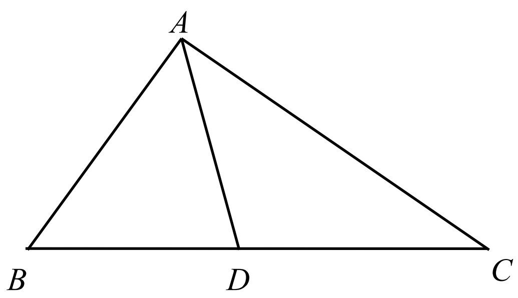
(2), $xy = 1 + z^2 > 0$; together with (1) we have $x > 0$ and $y > 0$

by A.M. ≥ G.M. in (1) $x + y \ge 2\sqrt{xy} \Rightarrow 2 \ge 2\sqrt{1 + z^2}$

After simplification, $0 \ge z^2 \Rightarrow z = 0$

(3): $b = x + y + z = 2 + 0 = 2$

I1.3 In the figure, $BD = b$ cm, $DC = c$ cm and area of $\Delta ABD = \frac{1}{3} \times$area of ΔABC, find the value of c.



Let the common height be h cm

$\frac{1}{2} BD \times hcm = \frac{1}{3} \cdot \frac{1}{2} BC \times hcm$

$2 = \frac{1}{3}(2 + c) \Rightarrow c = 4$

I1.4 Suppose d is the number of positive factors of $500 + c$, find the value of d.

Reference 1993 HI8, 1994 FI3.2, 1997 HI3, 1998 HI10, 2002 FG4.1, 2005 FI4.4

$500 + c = 504 = 2^3 \times 3^2 \times 7$

A positive factor is in the form $2^i \times 3^j \times 7^k$, where $0 \le i \le 3$, $0 \le j \le 2$, $0 \le k \le 1$

The total number of positive factors are $(1 + 3)(1 + 2)(1 + 1) = 24$

Individual Event 2

I2.1 If $A(1, 3)$, $B(5, 8)$ and $C(29, a)$ are collinear, find the value of a.

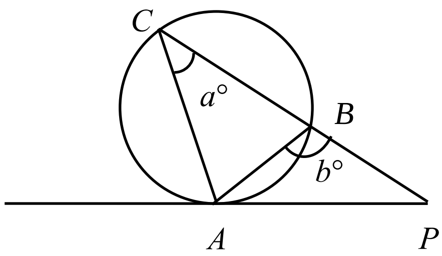
The slopes are equal: $\frac{8-3}{5-1}=\frac{a-8}{29-5}$

$\frac{a-8}{24}=\frac{5}{4}$

$\Rightarrow a - 8 = 30$

$a = 38$

I2.2 In the figure, PA touches the circle ABC at A, PBC is a straight line, $AB = PB$, $\angle ACB = a°$. If $\angle ABP = b°$, find the value of b.



$\angle BAP = a° = 38°$ ($\angle$ in alt. seg.)

$\angle BPA = 38°$ (base $\angle$s isos. Δ)

$38 + 38 + b = 180$ ($\angle$ sum of Δ)

$b = 104$

I2.3 If c is the minimum value of the quadratic function $y = x^2 + 4x + b$, find the value of c.

$y = x^2 + 4x + 104 = (x + 2)^2 + 100 \ge 100 = c$

I2.4 If $d = 1 - 2 + 3 - 4 + \cdots - c$, find the value of d.

Reference: 1991 FSI.1

$d = (1 - 2) + (3 - 4) + \cdots + (99 - 100)$

$= -1 - 1 - \cdots - 1$ (50 times)

$= -50$

Individual Event 3

I3.1 If $\{p, q\} = q \times a + p$ and $\{2, 5\} = 52$, find the value of a.

$\{2, 5\} = 5 \times a + 2 = 52$

$a = 10$

I3.2 If a, $\frac{37}{2}$, b is an arithmetic progression, find the value of b.

$\frac{a+b}{2} = \frac{37}{2}$

$b = 27$

I3.3 If $b^2 - c^2 = 200$ and $c > 0$, find the value of c.

$27^2 - c^2 = 200$

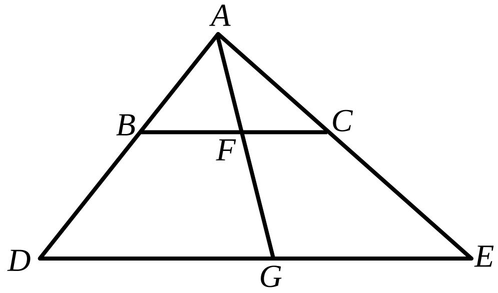
$c^2 = 729 - 200 = 529$

$c = 23$

Remark: Original question is: If $b^2 - c^2 = 200$, find the value of c.

$c = \pm 23$, c is not unique.

I3.4 Given that in the figure, BC // DE, $BC : DE = 10 : c$ and $AF : FG = 20 : d$, find the value of d.



By similar triangles, $AF : AG = AC : AE = BC : DE$

$20 : (20 + d) = 10 : 23$

$d = 26$

Individual Event 4

I4.1 Given that $\frac{10x-3y}{x+2y}=2$ and $p=\frac{y+x}{y-x}$, find the value of p.

$10x - 3y = 2(x + 2y)$

$8x = 7y$

$p=\frac{y+x}{y-x}$

$=\frac{8y+8x}{8y-8x}$

$=\frac{8y+7y}{8y-7y}=15$

I4.2 Given that $a \neq b$ and $ax = bx$. If $p + q = 19(a - b)^x$, find the value of q.

$a \neq b$ and $ax = bx \Rightarrow x = 0$

$p + q = 19(a - b)^x$

$\Rightarrow 15 + q = 19$

$q = 4$

I4.3 Given that the sum of q consecutive numbers is 222, and the largest of these consecutive numbers is r, find the value of r.

The smallest integer is $r - q + 1$

$\frac{q}{2}(r-q+1+r)=222$

$\Rightarrow 2(2r - 3) = 222$

$r = 57$

I4.4 If $\tan^2(r + s)° = 3$ and $0 \leq r + s \leq 90$, find the value of s.

$\tan^2(57 + s)° = 3$

$57 + s = 60$

$s = 3$

Individual Event 5

I5.1 If the sum of roots of $5x^2 + ax - 2 = 0$ is twice the product of roots, find the value of a.

$\alpha + \beta = 2\alpha\beta$

$-\frac{a}{5} = 2\left(-\frac{2}{5}\right)$

$a = 4$

I5.2 Given that $y = ax^2 - bx - 13$ passes through (3, 8), find the value of b.

$8 = 4(3)^2 - b(3) - 13$

$b = 5$

I5.3 If there are c ways of arranging b girls in a circle, find the value of c.

Reference: 2000 FG4.4, 2011 FI1.4

First arrange the 5 girls in a line, the number of ways = 5×4×3×2×1 = 120

Next, join the first girl and the last girl to form a circle. There are 5 repetitions.

The number of ways = c = 120 ÷ 5 = 24

I5.4 If $\frac{c}{4}$ straight lines and 3 circles are drawn on a paper, and d is the largest numbers of points of intersection, find the value of d.

For the 3 circles, there are 6 intersections.

If each straight line is drawn not passing through these intersections, it intersects the 3 circles at 6 other points. The 6 straight lines intersect each other at 1 + 2 + 3 + 4 + 5 points.

$\therefore$ d = the largest numbers of points of intersection = 6 + 6×6 + 15 = 57

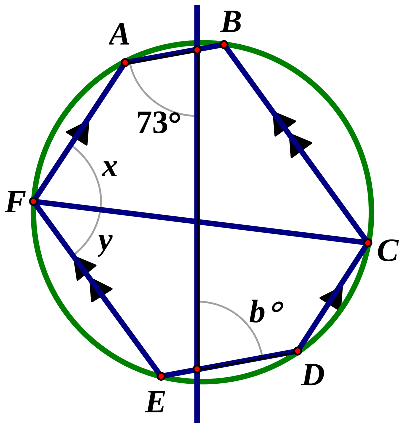
Sample Group Event

SG.1 If a is the smallest positive integer which gives remainder 1 when divided by 3 and is a multiple of 5, find the value of a. **(Reference: 1997 FG5.1)**

$a = 5k = 3m + 1$

The smallest possible $a = 10$.

SG.2 In the following diagram, $FA//DC$ and $FE//BC$. Find the value of b.



Join AD and CF.

Let $\angle CFE = x$, $\angle AFC = y$

$\angle BCF = x$ (alt. $\angle$s, $FE // BC$)

$\angle DCF = y$ (alt. $\angle$s, $FA // DC$)

$\angle BCD = x + y$

$\angle BAD = 180° - x - y = \angle ADE$ (opp. $\angle$ cyclic quad.)

$\therefore AB // ED$ (alt. $\angle$s eq.)

$b = 73$ (alt. $\angle$s $AB // ED$)

SG.3 If c is a 2 digit positive integer such that sum of its digits is 10 and product of its digit is 25, find the value of c. **(Reference: 1997 FG5.3)**

$c = 10x + y$, where $0 < x < 10$, $0 \le y < 10$.

$x + y = 10$

$xy = 25$

Solving these two equations gives $x = y = 5$; $c = 55$

SG.4 Let $S_1, S_2, \dots, S_{10}$ be the first ten terms of an A.P., which consists of positive integers.
If $S_1 + S_2 + \dots + S_{10} = 55$ and $(S_{10} - S_8) + (S_9 - S_7) + \dots + (S_3 - S_1) = d$, find d.
Reference: 1997 FG5.4

Let the general term be $S_n = a + (n - 1)t$

$$\frac{10}{2}[2a + (10-1)t] = 55$$

$\Rightarrow 2a + 9t = 11$

$\because$ a, t are positive integers, $a = 1$, $t = 1$

$d = (S_{10} - S_8) + (S_9 - S_7) + \dots + (S_3 - S_1)$
$= [a + 9t - (a + 7t)] + [a + 8t - (a + 6t)] + \dots + (a + 2t - a)$

$d = 2t + 2t + 2t + 2t + 2t + 2t + 2t + 2t$
$= 16t = 16$

Group Event 1

G1.1 If the area of a given sector $s = 4$ cm^2, the radius of this sector $r = 2$ cm and the arc length of this sector $A = p$ cm, find the value of p.

By the formula $A = \frac{1}{2}rs$, where A is the sector area, r is the radius and s is the arc length

$4 = \frac{1}{2}(2)p$

$p = 4$

G1.2 Given that $\frac{a}{2b+c} = \frac{b}{2c+a} = \frac{c}{2a+b}$ and $a + b + c \neq 0$. If $q = \frac{2b+c}{a}$, find the value of q.

Reference 2010 FG1.2

Let $\frac{a}{2b+c} = \frac{b}{2c+a} = \frac{c}{2a+b} = k$

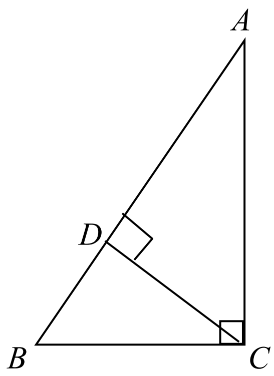
$a = (2b + c)k$; $b = (2c + a)k$; $c = (2a + b)k$

$a + b + c = (2b + c + 2c + a + 2a + b)k$

$a + b + c = (3a + 3b + 3c)k \Rightarrow k = \frac{1}{3}$

$q = \frac{2b+c}{a} = \frac{1}{k} = 3$

G1.3 Let ABC be a right-angled triangle, CD is the altitude on AB, $AC = 3$, $DB = \frac{5}{2}$, $AD = r$, find the value of r.



Reference: 1999 FG5.4, 2022 P1Q3

$AD = AC \cos A = \frac{3AC}{AB} = \frac{9}{\frac{5}{2} + AD}$

$\frac{5}{2}AD + AD^2 = 9$

$2AD^2 + 5AD - 18 = 0$

$(2AD + 9)(AD - 2) = 0$

$AD = r = 2$

G1.4 If $x^3 + px^2 + qx + 17 \equiv (x + 2)^3 + a$, find the value of a.

Reference: 1997 FG5.2

Compare the constant term: $17 = 8 + a$

$a = 9$

Group Event 2

G2.1 If $\frac{137}{a} = 0.1\dot{2}3\dot{4}$, find the value of a.

$\frac{137}{a} = 0.1\dot{2}3\dot{4} = 0.1 + \frac{234}{9990} = \frac{999+234}{9990} = \frac{1233}{9990} = \frac{137}{1110}$

$a = 1110$

G2.2 If $b = 1999\times19981998 - 1998\times19991999 + 1$, find the value of b.

Reference: 1996 FG9.3

$b = 1999\times1998\times1001 - 1998\times1999\times1001 + 1 = 1$

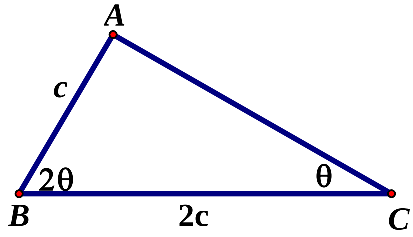
G2.3 If the parametric equation $\begin{cases} x = \sqrt{3-t^2} \\ y = t-3 \end{cases}$ can be transformed into $x^2 + y^2 + cx + dy + 6 = 0$, find the values of c and d.

$(1)^2 + (2)^2 : x^2 + y^2 = -6t + 12 = -6(y + 3) + 12$

$c = 0$, $d = 6$

Group Event 3

G3.1 In ΔABC, $\angle ABC = 2\angle ACB$, $BC = 2AB$.
If $\angle BAC = a°$, find the value of a.



Reference: 2001 HG8

Let $\angle ACB = \theta$, $\angle ABC = 2\theta$ (given)

$AB = c$, $BC = 2c$

$\angle BAC = 180° - \theta - 2\theta$ ($\angle$s sum of Δ)

By sine formula, $\frac{c}{\sin\theta} = \frac{2c}{\sin(180° - 3\theta)}$

$\sin 3\theta = 2\sin\theta$

$3\sin\theta - 4\sin^3\theta = 2\sin\theta$

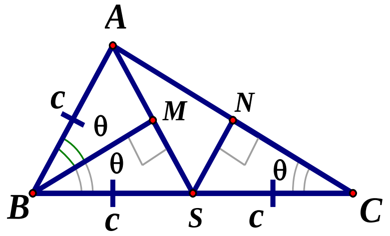
$4\sin^2\theta - 1 = 0$

$\sin\theta = \frac{1}{2}$; $\theta = 30°$, $\angle BAC = 180° - 3\theta = 90°$; $a = 90$

Method 2 Let $\angle ACB = \theta$, $\angle ABC = 2\theta$ (given)

Let S be the mid-point of BC.

Let N and M be the feet of perpendiculars drawn from S on AC and B from AS respectively.



$\Delta BSM \cong \Delta BAM$ (RHS)

$\angle RQN = \theta = \angle SQN$ (corr. $\angle$s, $\cong \Delta$'s)

$\Delta CSN \cong \Delta BSM \cong \Delta BAM$ (AAS)

$NS = MS = AM$ (corr. sides $\cong \Delta$'s)

$\sin \angle NAS = \frac{NS}{AS} = \frac{1}{2}$; $\angle NAS = 30°$;

$\angle ASN = 60°$ ($\angle$s sum of ΔASN)

$90° - \theta + 60° + 90° - \theta = 180°$ (adj. $\angle$s on st. line BSC)

$\theta = 30°$

$\angle BAC = 180° - 3\theta = 90°$ ($\angle$s sum of ΔABC)

$a = 90$

G3.2 Given that $x + \frac{1}{x} = \sqrt{2}$, $\frac{x^2}{x^4 + x^2 + 1} = b$, find the value of b.

$\left(x + \frac{1}{x}\right)^2 = 2 \Rightarrow x^2 + 2 + \frac{1}{x^2} = 2$

$\Rightarrow x^2 + \frac{1}{x^2} = 0$ (remark: x is a complex number)

$b = \frac{x^2}{x^4 + x^2 + 1} = \frac{1}{x^2 + 1 + \frac{1}{x^2}} = 1$

G3.3 If the number of positive integral root(s) of the equation $x + y + 2xy = 141$ is c, find the value of c.

$2x + 2y + 4xy = 282 \Rightarrow 2x + 2y + 4xy + 1 = 283$, which is a prime number

$(2x + 1)(2y + 1) = 1\times 283$

$2x + 1 = 1$, $2y + 1 = 283$ (or $2x + 1 = 283$, $2y + 1 = 1$)

Solving the above equations, there is no positive integral roots.

$c = 0$

G3.4 Given that $x + y + z = 0$, $x^2 + y^2 + z^2 = 1$ and $d = 2(x^4 + y^4 + z^4)$, find the value of d.

Let $x + y + z = 0$ …… (1), $x^2 + y^2 + z^2 = 1$ …… (2)

From (1), $(x + y + z)^2 = 0$

$\Rightarrow x^2 + y^2 + z^2 + 2(xy + yz + zx) = 0$

Sub. (2) into the above equation, $xy + yz + zx = -\frac{1}{2}$ …… (3)

From (3), $(xy + yz + zx)^2 = \frac{1}{4}$

$\Rightarrow x^2y^2 + y^2z^2 + z^2x^2 + 2xyz(x + y + z) = \frac{1}{4}$

Sub. (1) into the above equation, $x^2y^2 + y^2z^2 + z^2x^2 = \frac{1}{4}$ …… (4)

From (2), $(x^2 + y^2 + z^2)^2 = 1$

$\Rightarrow x^4 + y^4 + z^4 + 2(x^2y^2 + y^2z^2 + z^2x^2) = 1$

Sub. (4) into the above equation, $x^4 + y^4 + z^4 = \frac{1}{2}$ …… (5)

Sub. (5) into $d \Rightarrow d = 2(x^4 + y^4 + z^4) = 2\times\frac{1}{2} = 1$

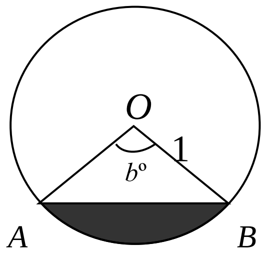
Group Event 4

G4.1 If $0.\dot{1}+0.0\dot{2}+0.00\dot{3}+\cdots+0.00000000\dot{9}=a$, find the value of a (Give your answer in decimal)

$$a=\frac{1}{9}+\frac{2}{90}+\frac{3}{900}+\cdots+\frac{9}{900000000}=\frac{100000000+20000000+3000000+\cdots+9}{900000000}$$

$$a=\frac{123456789}{900000000}=\frac{13717421}{100000000}=0.13717421$$

G4.2 The circle in the figure has centre O and radius 1, A and B are points on the circle. Given that $\frac{\text{Area of shaded part}}{\text{Area of unshaded part}}=\frac{\pi-2}{3\pi+2}$ and $\angle AOB=b°$, find the value of b.



$$\frac{\text{Area of shaded part}}{\text{Area of the circle}}=\frac{\pi-2}{\pi-2+3\pi+2}=\frac{\pi-2}{4\pi}$$

$$\frac{\pi(1)^2\cdot\frac{b}{360}-\frac{1}{2}(1)^2\sin b°}{\pi(1)^2}=\frac{\pi-2}{4\pi}\Rightarrow\frac{\pi b}{90}-2\sin b°=\pi-2;\ b=90$$

G4.3 A sequence of figures S_0, S_1, S_2, … are constructed as follows. S_0 is obtained by removing the middle third of [0,1] interval; S_1 by removing the middle third of each of the two intervals in S_0; S_2 by removing the middle third of each of the four intervals in S_1; S_3, S_4, … are obtained similarly. Find the total length c of the intervals removed in the construction of S_5 (Give your answer in fraction).

|――――――( )――――――| S_0

0 $\frac{1}{3}$ $\frac{2}{3}$ 1

|――( )――( )――( )――| S_1

0 $\frac{1}{9}$ $\frac{2}{9}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{7}{9}$ $\frac{8}{9}$ 1

|-( )-( )-( )-( )-( )-( )-( )-| S_2

The total length in $S_0=\frac{2}{3}$

The total length in $S_1=4\times\frac{1}{9}=\frac{4}{9}$

The total length in $S_2=8\times\frac{1}{27}=\frac{8}{27}$

Deductively, the total length in $S_5=2^6\times\frac{1}{3^6}=\frac{64}{729}$

The total length removed in $S_5=1-\frac{64}{729}=\frac{665}{729}$

G4.4 All integers are coded as shown in the following table. If the sum of all integers coded from 101 to 200 is d, find the value of d.

Integer	⋯	⋯	–3	–2	–1	0	1	2	3	⋯	⋯
Code	⋯	⋯	7	5	3	1	2	4	6	⋯	⋯

Sum of integers code as 102, 104, ⋯ , 200 is 51 + 52 + ⋯ + 100

Sum of integers code as 101, 103, ⋯ , 199 is –50 – 51 – ⋯ – 99

Sum of all integers = 1 + 1 + ⋯ + 1 (50 times) = 50

Group Event 5

G5.1 If 1×2×3 + 2×3×4 + 3×4×5 +…+ 10×11×12 = a, find the value of a.

$$a=\frac{1}{4}n(n+1)(n+2)(n+3)$$

$$=\frac{1}{4}10(11)(12)(13)=4290$$

G5.2 Given that $5^x + 5^{-x} = 3$. If $5^{3x} + 5^{-3x} = b$, find the value of b.

Reference: 1983 FG7.3, 1996FI1.2, 2010 FI3.2

$(5^x + 5^{-x})^2 = 9$

$\Rightarrow 5^{2x} + 2 + 5^{-2x} = 9$

$\Rightarrow 5^{2x} + 5^{-2x} = 7$

$b = 5^{3x} + 5^{-3x}$

$= (5^x + 5^{-x})(5^{2x} - 1 + 5^{-2x})$

$= 3(7 - 1) = 18$

G5.3 Given that the roots of equation $x^2 + mx + n = 0$ are 98 and 99 and $y = x^2 + mx + n$. If x takes on the values of 0, 1, 2, ⋯ , 100, then there are c values of y that can be divisible by 6.

Find the value of c.

$m = -98 - 99 = -197$; $n = 98\times99 = 49\times33\times6$, which is divisible by 6

$y = x^2 - 197x + 98\times99$

$= x^2 + x - 198x + 49\times33\times6$

$= x(x + 1) - 6(33x + 49\times33)$

If y is divisible by 6, then $x(x + 1)$ is divisible by 6

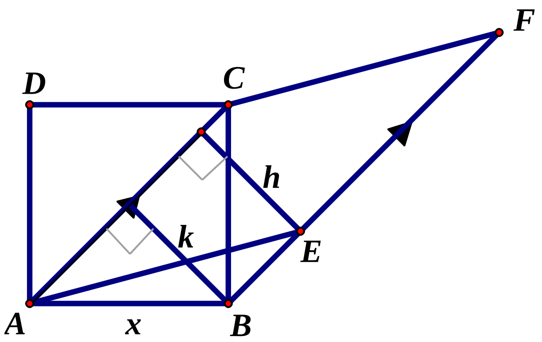
One of x, $x + 1$ must be even. If it is divisible by 6, then one of x, $x + 1$ must be divisible by 3.

We count the number of possible x for which y cannot be divisible by 6

These x may be 1, 4, 7, 10, ⋯ , 97, 100; totally 34 possible x.

$c = 101 - 34 = 67$

G5.4 In the figure, $ABCD$ is a square, BF // AC, and $AEFC$ is a rhombus. If $\angle EAC = d°$, find the value of d.



Reference HKCEE Mathematics 1992 P2 Q54

From B and E draw 2 lines h, $k \perp AC$

$h = k$ ($\because BF$ // AC)

Let $AB = x$, $\angle CAB = 45°$

$$k = x\sin 45° = \frac{x}{\sqrt{2}} = h$$

$AC = x \div \cos 45°$

$= \sqrt{2}x = AE$ ($\because AEFC$ is a rhombus)

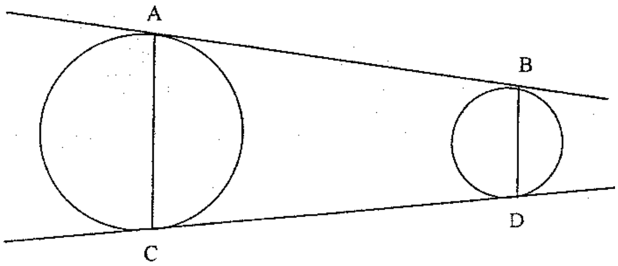
$$\sin\angle EAC = \frac{h}{AE}$$

$$=\frac{\frac{x}{\sqrt{2}}}{\sqrt{2}x}=\frac{1}{2}$$

$d = 30$

Group Spare Event

GS.1 In the figure, there are two common tangents. These common tangents meet the circles at points *A*, *B*, *C* and *D*. If AC =9 cm, BD = 3 cm, $\angle BAC = 60°$ and $AB = s$ cm, find the value of *s*.



Produce *AB* and *CD* to meet at *E*.

$AE = CE$, $BE = DE$ (tangent from ext. pt.)

ΔEAC and ΔEBD are isosceles triangles

$\angle ECA = \angle BAC = 60°$ (base ∠s isos. Δ)

$\angle AEC = 60°$ (∠ sum of Δ)

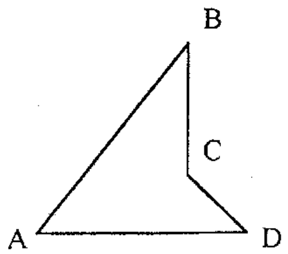
$\angle EBD = \angle EDB = 60°$ (∠ sum of Δ, base ∠s isos. Δ)

∴ ΔEAC and ΔEBD are equilateral triangles

$EB = BD = 3$ cm, $EA = AC = 9$ cm (sides of equilateral triangles)

$s = 9 - 3 = 6$

GS.2 In the figure, *ABCD* is a quadrilateral, where the interior angles ∠*A*, ∠*B* and ∠*D* are all equal to 45°. When produced, *BC* is perpendicular to *AD*. If $AC = 10$ and $BD = b$, find the value of *b*.



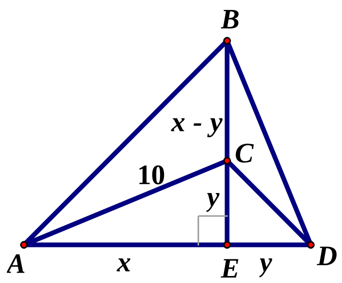
reflex $\angle BCD = 360° - 45° - 45° - 45° = 225°$ (∠ sum of polygon)

$\angle BCD = 360° - 225° = 135°$ (∠s at a point)

Produce *BC* to meet *AD* at *E*, $\angle AEB = 90°$ (given)

$\angle BAE = 45° = \angle ABE$ (given)

ΔABE and ΔCDE are right angled isosceles triangles



Let $AE = x$, $DE = y$, then $BE = x$, $CE = y$, $BC = x - y$

In ΔACE, $x^2 + y^2 = 10^2$ … (1) (Pythagoras' theorem)

$CD = \sqrt{y^2 + y^2} = \sqrt{2}y$ (Pythagoras' theorem)

Apply cosine rule on ΔBCD

$BD^2 = (x - y)^2 + 2y^2 - 2(x - y)\sqrt{2}y \cos 135°$

$BD^2 = x^2 - 2xy + y^2 + 2y^2 + 2(x - y)y = x^2 + y^2 = 10^2$

$\Rightarrow BD = b = 10$

GS.3 If $\log_c 27 = 0.75$, find the value of *c*.

$c^{0.75} = 27$

$\Rightarrow c = \left(3^3\right)^{\frac{4}{3}} = 81$

GS.4 If the mean, mode and median of the data 30, 80, 50, 40, *d* are all equal, find the value of *d*.

Mean $= \dfrac{30 + 80 + 50 + 40 + d}{5} = 40 + \dfrac{d}{5}$ = mode

By trial and error, $d = 50$