## Individual Events

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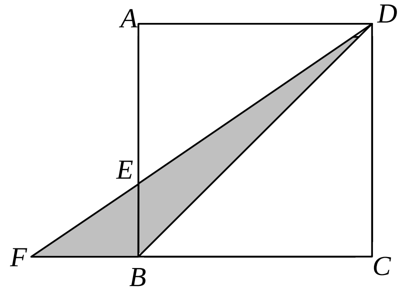
|---|---|---|---|---|---|---|---|---|---|---|
| **I1** | ***P*** | 40 | **I2** | ***P*** | $\frac{99}{100}$ | **I3** | ***P*** | 12 | **I4** | ***P*** | 4 |
| | ***Q*** | 72 | | ***Q*** | 1 | | ***Q*** | 1 | | ***Q*** | 8 |
| | ***R*** | 648 | | ***R*** | 3 | | ***R*** | 615 | | ***R*** | 4 |
| | ***S*** | 40.5 | | ***S*** | $\frac{1}{12}$ | | ***S*** | 60 | | ***S*** | 10 |

## Group Events

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **G1** | ***a*** | 21 | **G2** | ***a*** | 24 | **G3** | ***a*** | 2005 | **G4** | ***a*** | 4032 |
| | ***b*** | 2.5 | | ***b*** | 52 | | ***b*** | 2 | | ***b*** | 2 |
| | ***c*** | 19 | | ***c*** | 2005003 | | ***c*** | 649 | | ***c*** | 1 |
| | ***d*** | 300° | | ***d*** | 3 | | ***d*** | 8 | | ***d*** | 2 |

### Individual Event 1

**I1.1** In the following figure, $ABCD$ is a square of length 10 cm. $AEB$, $FED$ and $FBC$ are straight lines. The area of $\Delta AED$ is larger than that of $\Delta FEB$ by 10 $\text{cm}^2$. If the area of $\Delta DFB$ is $P$ $\text{cm}^2$, find the value of $P$.




Let the area of $\Delta BDE$ be $x$.

Then area of $\Delta AED + x$ – (area of $\Delta BEF + x$) = 10

area of $\Delta ABD$ – area of $\Delta BDF$ = 10

$\frac{1}{2}\cdot 10\times 10$ – area of $\Delta BDF$ = 10

area of $\Delta BDF$ = 40

**Method 2**

Area of $\Delta ADE$ – area of $\Delta BFE$ = 10 (given)

$\Rightarrow$ Area of $\Delta ADE$ + area of $\Delta AEF$ – area of $\Delta BFE$ – area of $\Delta AEF$ = 10

$\Rightarrow$ Area of $\Delta ADF$ – area of $\Delta AFB$ = 10

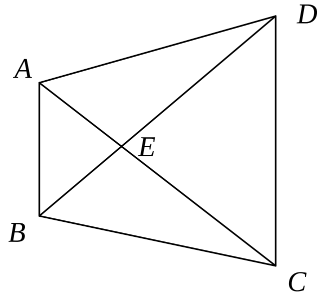
$\Rightarrow$ $\frac{1}{2}\cdot 10\times 10$ – area of $\Delta DFB$ = 10 $\Rightarrow$ Area of $\Delta DFB$ = 50 – 10 = 40

**I1.2** Workman A needs 90 days to finish a task independently while workman B needs $Q$ days for the same task. If they only need $P$ days to finish the task when working together, find the value of $Q$.

$\frac{1}{90}+\frac{1}{Q}=\frac{1}{40}$

$Q = 72$

**I1.3** In the following figure, $AB//CD$, the area of trapezium $ABCD$ is $R$ $\text{cm}^2$. Given that the areas of $\Delta ABE$ and $\Delta CDE$ are $Q$ $\text{cm}^2$ and $4Q$ $\text{cm}^2$ respectively, find the value of $R$.




**Reference: 1993 HI2, 1997 HG3, 2000 FI2.2, 2004 HG7, 2010HG4, 2013 HG2**

It is easy to show that $\Delta ABE \sim \Delta CDE$ (equiangular)

$Q : 4Q = (AB)^2 : (CD)^2 \Rightarrow AB : CD = 1 : 2$

$AE : EC = AE : EC = BE : ED = 1 : 2$ (ratio of sides, ~ Δ's)

$S_{\Delta AEB} : S_{\Delta AED} = BE : ED = 1 : 2$ (the 2 Δs have the same heights)

$S_{\Delta AED} = 2Q$

$S_{\Delta AEB} : S_{\Delta BEC} = AE : EC = 1:2$ (the 2 Δs have the same heights)

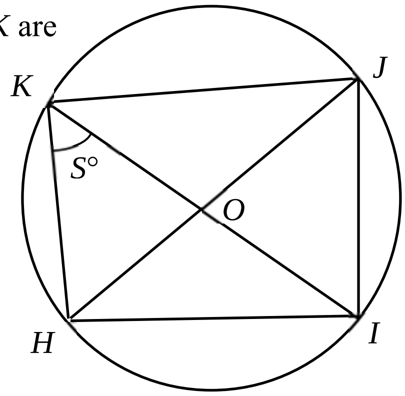
$S_{\Delta BEC} = 2Q$

$S_{ABCD} = Q + 4Q + 2Q + 2Q = 9Q = 648$

$R = 648$

**I1.4** In the following figure, $O$ is the centre of the circle, $HJ$ and $IK$ are diameters and $\angle HKI = S°$.

Given that $\angle HKI + \angle HOI + \angle HJI = \frac{1}{4}R°$, find the value of $S$.




$S°+2S°+S° = \frac{1}{4}\times 648°$

$\Rightarrow 4S°=162°$ ($\angle$ at centre = 2$\angle$ at $\odot^{ce}$)

$S = 40.5$

**Individual Event 2**

**I2.1** Given that $P = \frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\cdots+\frac{1}{99\times 100}$, find the value of $P$.

$P = 1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\cdots+\frac{1}{99}-\frac{1}{100}$

$= 1-\frac{1}{100}=\frac{99}{100}$

**I2.2** Given that $99Q = P\times(1+\frac{99}{100}+\frac{99^2}{100^2}+\frac{99^3}{100^3}+\ldots)$, find the value of $Q$.

$99Q = \frac{99}{100}\times(1+\frac{99}{100}+\frac{99^2}{100^2}+\frac{99^3}{100^3}+\ldots)$

$= \frac{99}{100}\times\frac{1}{1-\frac{99}{100}} = 99$

$Q = 1$

**I2.3** Given that $x$ and $R$ are real numbers and $\frac{2x^2+2Rx+R}{4x^2+6x+3}\le Q$ for all $x$, find the maximum value of $R$.

$4x^2 + 6x + 3 = (2x + 1.5)^2 + 0.75 > 0$

$\frac{2x^2+2Rx+R}{4x^2+6x+3}\le 1$

$2x^2 + 2Rx + R \le 4x^2 + 6x + 3$

$2x^2 + 2(3 - R)x + 3 - R \ge 0$

$\Delta \le 0$

$(3 - R)^2 - 2(3 - R) \le 0$

$(3 - R)(1 - R) \le 0$

$1 \le R \le 3$

The maximum value of $R = 3$

**I2.4** Given that $S = \log_{144}\sqrt[R]{2} + \log_{144}\sqrt[2R]{R}$, find the value of $S$.

$S = \frac{\frac{1}{3}\log 2}{\log 144}+\frac{\frac{1}{6}\log 3}{\log 144}=\frac{2\log 2+\log 3}{6\log 144}=\frac{\log 12}{6\log 12^2}=\frac{\log 12}{12\log 12}=\frac{1}{12}$

**Method 2**

$S = \log_{144}\sqrt[R]{2} + \log_{144}\sqrt[2R]{R}$

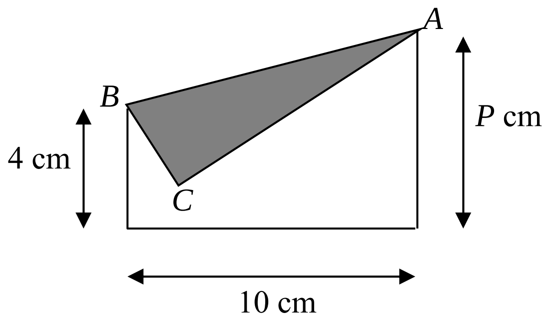
$= \log_{144}(\sqrt[3]{2}\cdot\sqrt[6]{3})$

$= \log_{144}(\sqrt[6]{12})$

$= \log_{144}(\sqrt[12]{144}) = \frac{1}{12}$

**Individual Event 3**

**I3.1** A rectangular piece of paper is folded into the following figure. If the area of $\triangle ABC$ is $\frac{1}{3}$ of the area of the original rectangular piece of paper, find the value of $P$.




$BC = P - 4$, $AC = 10$, $\angle ACB = 90°$

$$\frac{(P-4)\cdot 10}{2} = \frac{1}{3}\times P\times 10$$

$\Rightarrow P = 12$

**I3.2** If $Q$ is the positive integral solution of the equation $\frac{P}{2}(4^x + 4^{-x}) - 35(2^x + 2^{-x}) + 62 = 0$, find the value of $Q$.

Let $t = 2^x + 2^{-x}$, then $t^2 = 4^x + 4^{-x} + 2$

$\Rightarrow 4^x + 4^{-x} = t^2 - 2$

The equation becomes $6(t^2 - 2) - 35t + 62 = 0$

$6t^2 - 35t + 50 = 0$

$(2t - 5)(3t - 10) = 0$

$t = \frac{5}{2}$ or $\frac{10}{3}$

$2^x + 2^{-x} = \frac{5}{2}$ or $2^x + 2^{-x} = \frac{10}{3}$

$2^x + \frac{1}{2^x} = \frac{5}{2}$ or $2^x + \frac{1}{2^x} = \frac{10}{3}$

$2(2^x)^2 + 2 = 5(2^x)$ or $3(2^x)^2 + 3 = 10(2^x)$

$2(2^x)^2 - 5(2^x) + 2 = 0$ or $3(2^x)^2 - 10(2^x) + 3 = 0$

$(2\cdot 2^x - 1)(2^x - 2) = 0$ or $(3\cdot 2^x - 1)(2^x - 3) = 0$

$2^x = \frac{1}{2}, 2, \frac{1}{3}$ or $3$

For positive integral solution $x = 1$; $Q = 1$

**I3.3** Let $[a]$ be the largest integer not greater than a. For example, $[2.5] = 2$.

If $R = [\sqrt{1}] + [\sqrt{2}] + \cdots + [\sqrt{99Q}]$, find the value of $R$.

$$R = [\sqrt{1}] + [\sqrt{2}] + \cdots + [\sqrt{99}] = 1+1+1+\underbrace{2+\cdots+2}_{5\text{ times}}+\underbrace{3+\cdots+3}_{7\text{ times}}+\cdots+\underbrace{9+\cdots+9}_{19\text{ times}}$$

$R = 3\times1 + 5\times2 + 7\times3 + \cdots + 19\times9$

$R = (2\times1+1)\times1 + (2\times2+1)\times2+(2\times3+1)\times3 + \cdots + (2\times9+1)\times9$

$R = 2\times1^2 + 1 + 2\times2^2 + 2 + 2\times3^2 + 3 + \cdots + 2\times9^2 + 9$

$R = 2\times(1^2 + 2^2 + 3^2 + \cdots + 9^2) + (1 + 2 + 3 + \cdots + 9)$

$$R = 2\times\frac{1}{6}\cdot 9(9+1)(2\times 9+1)+\frac{(1+9)9}{2} = 3\times10\times19 + 45 = 570 + 45 = 615$$

**I3.4** In a convex polygon, other than the interior angle $A$, the sum of all the remaining interior angles is equal to $4R°$. If $\angle A = S°$, find the value of $S$.

**Reference: 1989 HG2, 1990 FG10.3-4, 1992 HG3, 2013 HI6**

$4\times615 + S = 180\times(n - 2)$

$S = 180(n - 2) - 2460$

$\because$ The polygon is convex

$\therefore S < 180$. $S = 180(14) - 2460 = 60$

**Individual Event 4**

**I4.1** Given that $f(x) = (x^2 + x - 2)^{2002} + 3$ and $f\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right) = P$, find the value of $P$.

$x = \frac{\sqrt{5}}{2}-\frac{1}{2}$

$\Rightarrow 2x = \sqrt{5} - 1$

$\Rightarrow (2x + 1) = \sqrt{5}$

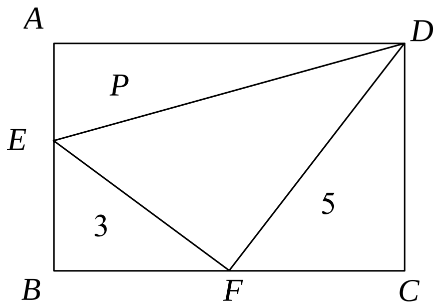
$\Rightarrow (2x + 1)^2 = 5$

$\Rightarrow 4x^2 + 4x - 4 = 0$

$\Rightarrow x^2 + x = 1$

$f(x) = (x^2 + x - 2)^{2002} + 3 = (1 - 2)^{2002} + 3 = 1 + 3 = 4$

**I4.2** In the following figure, $ABCD$ is a rectangle. $E$ and $F$ are points on $AB$ and $BC$ respectively. The areas of triangles $AED$, $EBF$ and $FCD$ are $P$, 3 and 5 respectively. If the area of $\Delta EFD$ is $Q$, find the value of $Q$.




Let $AE = x$, $CF = y$, $AD = b$, $CD = a$.

Then $BE = a - x$, $BF = b - y$

Given the area of $\Delta ADE = 4 \Rightarrow bx = 8$ ⋯⋯ (1)

the area of $\Delta CDF = 5 \Rightarrow ay = 10$ ⋯⋯(2)

The area of $\Delta BEF = 3 \Rightarrow (a - x)(b - y) = 6$

$\Rightarrow ab - bx - ay + xy = 6$ ⋯⋯ (3)

Sub. (1), (2) into (3) $ab - 8 - 10 + xy = 6$

Sub. (1), (2) into the equation again: $ab - 18 + \frac{80}{ab} = 6$

Solving for $ab$, $ab = 20$ or 4 (rejected)

The area of $\Delta DEF = 20 - 3 - 4 - 5 = 8$

$Q = 8$

**I4.3** It is given that $x$ and $y$ are positive integers. If the number of solutions $(x, y)$ of the inequality $x^2 + y^2 \le Q$ is $R$, find the value of $R$. **(Reference: 2007 FG1.2)**

$x^2 + y^2 \le 8$

$\Rightarrow (x, y) = (1, 1), (1, 2), (2, 1), (2, 2)$

$R = 4$

**I4.4** It is given that $\alpha$ and $\beta$ are roots of the equation $x^2 - ax + a - R = 0$, where $a$ is real. If the minimum value of $(\alpha + 1)^2 + (\beta + 1)^2$ is $S$, find the value of $S$.

$x^2 - ax + a - 4 = 0$; $\alpha + \beta = a$, $\alpha\beta = a - 4$

$(\alpha+1)^2 + (\beta+1)^2 = \alpha^2 + 2\alpha + 1 + \beta^2 + 2\beta + 1$

$= (\alpha + \beta)^2 - 2\alpha\beta + 2(\alpha + \beta) + 2$

$= a^2 - 2(a - 4) + 2a + 2 = a^2 + 10 \ge 10$

The minimum value is $S = 10$.

**Group Event 1**

**G1.1** Assume that the curve $x^2 + 3y^2 = 12$ and the straight line $mx + y = 16$ intersect at only one point. If $a = m^2$, find the value of $a$.

Sub. $y = 16 - mx$ into $x^2 + 3y^2 = 12$

$\Rightarrow x^2 + 3(16 - mx)^2 = 12$

$x^2 + 3(256 - 32mx + m^2x^2) = 12$

$\Rightarrow (1 + 3m^2)x^2 - 96mx + 756 = 0$

The straight line is a tangent $\Rightarrow \Delta = (-96m)^2 - 4(1 + 3m^2)756 = 0$

$576m^2 - 189(1 + 3m^2) = 0$

$\Rightarrow 64m^2 - 21(1 + 3m^2) = 0$

$\Rightarrow a = m^2 = 21$

**G1.2** It is given that $x + y = 1$ and $x^2 + y^2 = 2$. If $x^3 + y^3 = b$, find the value of $b$.

**Reference: 2011 FI2.2**

$(x + y)^2 = 1 \Rightarrow x^2 + y^2 + 2xy = 1$

$\Rightarrow 2 + 2xy = 1$

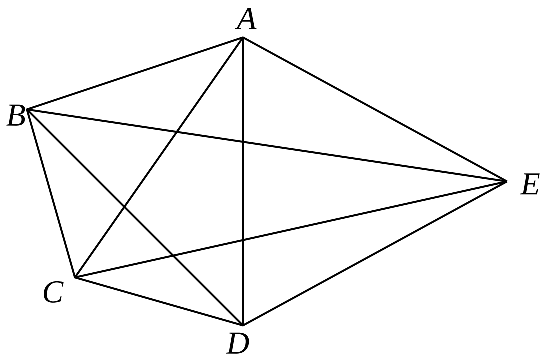
$\Rightarrow xy = -\frac{1}{2}$

$b = x^3 + y^3$

$= (x + y)(x^2 + y^2 - xy)$

$= 1(2 + \frac{1}{2}) = \frac{5}{2}$

**G1.3** In the following figure, $AC = AD = AE = ED = DB$ and $\angle BEC = c°$. Given that $\angle BDC = 26°$ and $\angle ADB = 46°$, find the value of $c$.




$\Delta ADE$ is an equilateral triangle.

$\angle DAE = \angle ADE = \angle AED = 60°$

$\because BD = DE$ and $\angle BDE = 46° + 60° = 106°$

$\therefore \angle BED = (180° - 106°)\div 2 = 37°$ ($\angle$ sum of $\Delta$)

$\angle AEB = 60° - 37° = 23°$

$\angle ADC = 26° + 46° = 72°$

$\because AC = AD$ and $\angle ADC = 72° = \angle ACD$ (base $\angle$, isos. $\Delta$)

$\therefore \angle CAD = 180° - 72°\times 2 = 36°$ ($\angle$s sum of $\Delta$)

$\because AC = AE$ and $\angle CAE = 36° + 60° = 96°$

$\therefore \angle AEC = (180° - 96°)\div 2 = 42°$ ($\angle$s sum of $\Delta$)

$\angle CED = 60° - 42° = 18°$

$\angle BCE = 60° - 18° - 23° = 19°$

$c = 19$

**G1.4** It is given that $4 \cos^4 \theta + 5 \sin^2 \theta - 4 = 0$, where $0° < \theta < 360°$. If the maximum value of $\theta$ is $d$, find the value of $d$.

$4 \cos^4 \theta + 5 \sin^2 \theta - 4 = 0 \Rightarrow 4 \cos^4 \theta + 5(1 - \cos^2 \theta) - 4 = 0 \Rightarrow 4 \cos^4 \theta - 5 \cos^2 \theta + 1 = 0$

$(4\cos^2 \theta - 1)(\cos^2 \theta - 1) = 0$

$\cos^2 \theta = \frac{1}{4}$ or 1

$\Rightarrow \cos \theta = \frac{1}{2}, -\frac{1}{2}$, 1 or $-1$.

$\theta = 60°, 300°, 120°, 180°, 240°$.

The maximum value of $\theta = 300°$

$d = 300°$

**Group Event 2**

**G2.1** It is given that the lengths of the sides of a triangle are 6, 8, and 10.

If the area of the triangle is $a$, find the value of $a$.

$6^2 + 8^2 = 36 + 64 = 100 = 10^2$

It is a right angled triangle.

The area of the triangle = $6 \times 8 \div 2 = 24$

$a = 24$

**G2.2** Given that $f\left(x+\frac{1}{x}\right)=x^3+\frac{1}{x^3}$ and f(4) = $b$, find the value of $b$ .

**Reference: 1987 FG8.2, 2002 HI10**

Let $y = x+\frac{1}{x}$

$y^2 - 2 = x^2+\frac{1}{x^2}$

$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)\left(x^2-1+\frac{1}{x^2}\right)$

$= y(y^2 - 3) = y^3 - 3y$

$f(y) = y^3 - 3y$

$b = f(4) = 4^3 - 3(4) = 52$

**G2.3** Given that $2002^2 - 2001^2 + 2000^2 - 1999^2 + \cdots + 4^2 - 3^2 + 2^2 - 1^2 = c$, find the value of $c$.

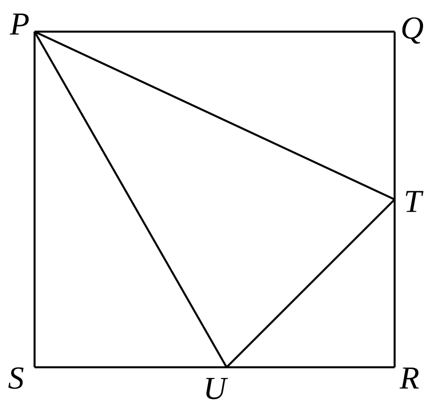
**Reference: 1997 HI5, 2004 HI1, 2015 FI3.2, 2015 FG4.1**

$c = (2002+ 2001)(2002 - 2001) + (2000 + 1999)(2000 - 1999) +\cdots+ (4+3)(4-3) + (2+1)(2-1)$

$c = 4003 + 3999 + \cdots + 7 + 3$

$=\frac{4003+3}{2}\times 1001 = 2005003$

**G2.4** $PQRS$ is a square, $PTU$ is an isosceles triangle, and $\angle TPU = 30°$. Points $T$ and $U$ lie on $QR$ and $RS$ respectively. The area of $\Delta PTU$ is 1. If the area of $PQRS$ is $d$, find the value of $d$.




Let $PT = a = PU$

$\frac{1}{2}a^2 \sin 30^\circ = 1$

$\Rightarrow a = 2$

$\Delta PSU \cong \Delta PQT$ (RHS)

Let $PS = x = PQ$; $SU = y = QT$

$\angle SPU = \angle QPT = 30°$ (corr. $\angle$s $\cong \Delta$)

$x = PU \cos 30° = 2\times\frac{\sqrt{3}}{2}=\sqrt{3}$

$d$ = area of $PQRS = \sqrt{3}^2 = 3$

**Group Event 3**

**G3.1** If $\frac{2002^3 + 4\times 2002^2 + 6006}{2002^2 + 2002} = a$, find the value of $a$.

$a = \frac{2002(2002^2 + 4\times 2002 + 3)}{2002(2002+1)}$

$= \frac{(2002+1)(2002+3)}{2002+1}$

$= 2005$

**G3.2** It is given that the real numbers $x$ and $y$ satisfy the relation $y = \frac{x}{2x-1}$.

If the minimum value of $\frac{1}{x^2} + \frac{1}{y^2}$ is $b$, find the value of $b$.

$\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{x^2} + \frac{(2x-1)^2}{x^2}$

$= \frac{4x^2 - 4x + 2}{x^2}$

Let $T = \frac{4x^2 - 4x + 2}{x^2}$

$Tx^2 = 4x^2 - 4x + 2$

$(T-4)x^2 + 4x - 2 = 0$

$\Delta = 4^2 + 4\times 2(T-4) \ge 0$

$2 + T - 4 \ge 0$

$\Rightarrow T \ge 2$

The minimum value is 2

$b = 2$

**G3.3** Suppose two different numbers are chosen randomly from the 50 positive integers 1, 2, 3, … , 50, and the sum of these two numbers is not less than 50. If the number of ways of choosing these two numbers is $c$, find the value of $c$.

**Reference: 2011 FG2.2**

Possible combinations may be: (1, 49), (1, 50),

(2, 48), (2, 49), (2, 50),

(3, 47), (3, 48), (3, 49), (3, 50),

……………………………………

(24, 26), (24, 27), ⋯ , (24, 50),

(25, 26), (25, 27), ⋯ , (25, 50),

(26, 27), … , (26, 50)

…………………………

(49, 50)

Total number of combinations = (2 + 3 + ⋯ + 25) + 25 + (24 + 23 + ⋯ + 1)

= (1 + 2 + ⋯ + 24)×2 + 24 + 25

= 25×24 + 49 = 649

**G3.4** Given that $x - y = 1 + \sqrt{5}$, $y - z = 1 - \sqrt{5}$. If $x^2 + y^2 + z^2 - xy - yz - zx = d$, find the value of $d$.

$2d = (x-y)^2 + (y-z)^2 + (z-x)^2 = (1+\sqrt{5})^2 + (1-\sqrt{5})^2 + [(z-y)-(x-y)]^2$

$2d = 1 + 2\sqrt{5} + 5 + 1 - 2\sqrt{5} + 5 + [-1+\sqrt{5} - (1+\sqrt{5})]^2 = 12 + 4 = 16$

$d = 8$

**Group Event 4**

**G4.1** If $a$ is the sum of all the positive factors of 2002, find the value of $a$.

**Reference 1997 HI3, 1998 HI10**

$2002 = 2\times7\times11\times13$

The positive factors may be $2^a 7^b 11^c 13^d$, where $0 \le a, b, c, d \le 1$ are integers.

The sum of all positive factors are $(1 + 2)(1 + 7)(1 + 11)(1 + 13) = 3\times8\times12\times14 = 4032 = a$

**G4.2** It is given that $x > 0$, $y > 0$ and $\sqrt{x}(\sqrt{x}+\sqrt{y}) = 3\sqrt{y}(\sqrt{x}+5\sqrt{y})$. If $b = \dfrac{2x+\sqrt{xy}+3y}{x+\sqrt{xy}-y}$, find the value of $b$.

$\sqrt{x}(\sqrt{x}+\sqrt{y}) = 3\sqrt{y}(\sqrt{x}+5\sqrt{y})$

$\Rightarrow x + \sqrt{xy} = 3\sqrt{xy} + 15y$

$\Rightarrow x - 2\sqrt{xy} - 15y = 0$

$\left(\sqrt{x}+3\sqrt{y}\right)\left(\sqrt{x}-5\sqrt{y}\right) = 0$

$\Rightarrow \sqrt{x} = 5\sqrt{y} \Rightarrow x = 25y$

$$b = \frac{2x+\sqrt{xy}+3y}{x+\sqrt{xy}-y} = \frac{50y+\sqrt{25y^2}+3y}{25y+\sqrt{25y^2}-y} = \frac{58y}{29y} = 2$$

$b = 2$

**G4.3** Given that the equation $||x-2|-1| = c$ has only 3 integral solutions, find the value of $c$.

**Reference: 2005 FG4.2, 2009 HG9, 2012 FG4.2, 2017 FG1.2**

$|x-2| - 1 = \pm c$

$\Rightarrow |x-2| = 1\pm c$

In order that it has only 3 integral solutions

$c = 1$

**G4.4** If $d$ is the positive real root of the equation $\dfrac{1}{2}\left\{\dfrac{1}{2}\left[\dfrac{1}{2}\left(\dfrac{1}{2}x^2+2\right)+2\right]+2\right\} = 2$, find the value of $d$.

$$\frac{1}{2}\left\{\frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{2}x^2+2\right)+2\right]+2\right\} = 2$$

$$\Rightarrow \left\{\frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{2}x^2+2\right)+2\right]+2\right\} = 4$$

$$\Rightarrow \frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{2}x^2+2\right)+2\right] = 2$$

$$\left[\frac{1}{2}\left(\frac{1}{2}x^2+2\right)+2\right] = 4$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{2}x^2+2\right) = 2$$

$$\Rightarrow \frac{1}{2}x^2+2 = 4$$

$$\Rightarrow \frac{1}{2}x^2 = 2$$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm2$

$d$ = the positive real root = 2