20-21 Paper 1	**1**	0	**2**	52	**3**	259	**4**	$3\sqrt{3}$	**5**	$6+4\sqrt{3}$
	6	6	**7**	15	**8**	1003	**9**	728	**10**	59
	11	$\frac{1}{27}$	**12**	10	**13**	$\frac{30\sqrt{13}}{13}$	**14**	4561	**15**	–1.5
20-21 Paper 2	**1**	2	**2**	3	**3**	$\frac{8}{9}$	**4**	75	**5**	$\frac{1}{2}$ cm^2
	6	1011	**7**	100060004						

Paper 1

1. 已知 $W=a^b-b^a$，其中 $a\neq b\neq 0$。若 W 為一非負整數，求 W 的最小值。

Given that $W=a^b-b^a$, where $a\neq b\neq 0$. If W is a non-negative integer, find the least value of W.

Let $a=4$, $b=2$.

The least value of $W=4^2-2^4=0$

2. 求 2^{2021} 的最尾兩位數字。

Find the last two digits of 2^{2021}.

$2^2=04$, $2^3=08$, $2^4=16$, $2^5=32$, $2^6=64$, $2^7\equiv 28$, $2^8\equiv 56$, $2^9\equiv 12$, $2^{10}\equiv 24$, $2^{11}\equiv 48$, $2^{12}\equiv 96$, $2^{13}\equiv 92$, $2^{14}\equiv 84$, $2^{15}\equiv 68$, $2^{16}\equiv 36$, $2^{17}\equiv 72$, $2^{18}\equiv 44$, $2^{19}\equiv 88$, $2^{20}\equiv 76$, $2^{21}\equiv 52$, $2^{22}\equiv 04$

The patterns repeats itself for every 20 powers.

$2^{2021}=2^{2002}\times 2^{19}\equiv 04\times 88\equiv 52 \pmod{100}$

3. α 及 β 為方程 $x^2-7x+4=0$ 的根。求 $\alpha^3+\beta^3$ 的值。

α and β are the roots of the equation $x^2-7x+4=0$. Find the value of $\alpha^3+\beta^3$.

$\alpha+\beta=7$, $\alpha\beta=4$

$$\begin{aligned}\alpha^3+\beta^3&=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)\\&=(\alpha+\beta)[(\alpha+\beta)^2-3\alpha\beta]\\&=7\times(7^2-3\times 4)\\&=259\end{aligned}$$

4. 求 $8\cos^2 15°\cos^2 30°-8\sin^2 15°\cos^2 30°$ 的值。

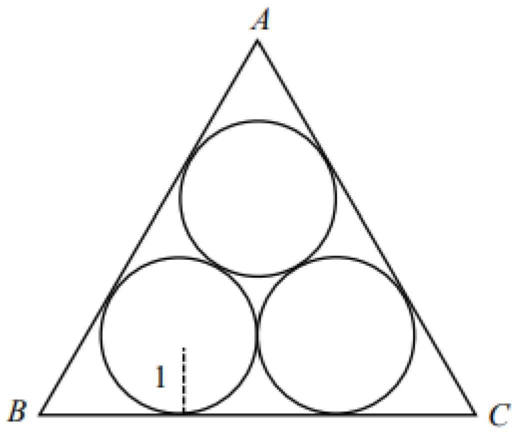
Find the value of $8\cos^2 15°\cos^2 30°-8\sin^2 15°\cos^2 30°$.

$$\begin{aligned}&8\cos^2 15°\cos^2 30°-8\sin^2 15°\cos^2 30°\\&=8(\cos^2 15°-\sin^2 15°)\cos^2 30°\\&=8\cos 30°\cos^2 30°\\&=8\times\frac{\sqrt{3}}{2}\times\left(\frac{\sqrt{3}}{2}\right)^2=3\sqrt{3}\end{aligned}$$

5. 在圖一中，三個單位圓位於一等邊三角形 ABC 內，使得每個圓均與另外兩圓及三角形的兩邊相切。求 ΔABC 的面積。

In Figure 1, three unit circles are placed inside an equilateral triangle ABC such that any circle is tangential to two sides of the triangle and to the other two circles. Find the area of ΔABC.

Reference: 1996 FG8.2, 1997 HG9



圖一 Figure 1

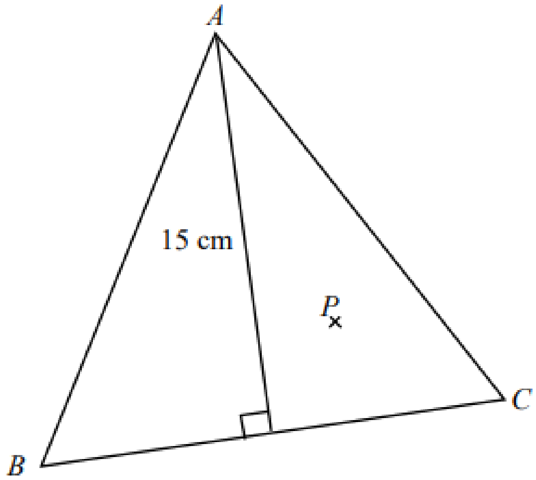
$AB=2+2\tan 60°=2+2\sqrt{3}$

$$\text{Area of }\Delta ABC=\frac{1}{2}\left(2+2\sqrt{3}\right)^2\sin 60°$$

$$=2\left(1+2\sqrt{3}+3\right)\cdot\frac{\sqrt{3}}{2}=6+4\sqrt{3}$$

6. 在圖二中，等邊三角形 ABC 的高為 15 cm。P 為 ΔABC 內的一點。從 P 與 AB、BC 和 AC 的垂直距離分別為 h cm、4 cm 和 5 cm。求 h 的值。

In Figure 2, the altitude of an equilateral triangle ABC is 15 cm. P is a point inside ΔABC. The perpendicular distances from P to AB, BC and AC are h cm, 4 cm and 5 cm respectively. Find the value of h.



圖二 Figure 2

Reference 1992 HG8, 2005 HG9, 2015 HG2

$BC = AB = CA$

$S_{\Delta APB} + S_{\Delta BPC} + S_{\Delta CPA} = S_{\Delta ABC}$

$$\frac{BC\times(4+5+h)}{2} = \frac{BC\times 15}{2}$$

$\Rightarrow h = 6$

7. p、q 及 r 為質數。若 $pqr = 7(p + q + r)$，求 $p + q + r$ 的值。

p, q and r are prime numbers. If $pqr = 7(p + q + r)$, find the value of $p + q + r$.

Without loss of generality, let $r = 7$, then $pq = p + q + 7$

$1 - p + pq - q = 8 \Rightarrow (p - 1)(q - 1) = 8$

$\because p, q$ are primes

$\Rightarrow p = 3, q = 5$ or vice versa

$\Rightarrow p + q + r = 15$

8. 求 $\dfrac{1001\times 1002}{\dfrac{1}{1+\dfrac{1}{1002}}+\dfrac{2}{2+\dfrac{2}{1002}}+\dfrac{3}{3+\dfrac{3}{1002}}+\cdots+\dfrac{1001}{1001+\dfrac{1001}{1002}}}$ 的值。

Find the value of $\dfrac{1001\times 1002}{\dfrac{1}{1+\dfrac{1}{1002}}+\dfrac{2}{2+\dfrac{2}{1002}}+\dfrac{3}{3+\dfrac{3}{1002}}+\cdots+\dfrac{1001}{1001+\dfrac{1001}{1002}}}$.

Let r be a positive integer such that $1 \le r \le 1001$.

$$\frac{r}{r+\dfrac{r}{1002}} = \frac{1}{1+\dfrac{1}{1002}} = \frac{1002}{1003}$$

$$\frac{1001\times 1002}{\dfrac{1}{1+\dfrac{1}{1002}}+\dfrac{2}{2+\dfrac{2}{1002}}+\dfrac{3}{3+\dfrac{3}{1002}}+\cdots+\dfrac{1001}{1001+\dfrac{1001}{1002}}} = \frac{1001\times 1002}{1001\times\dfrac{1002}{1003}} = 1003$$

9. 在 4000 和 7000 之間 4 個數位各不相同的偶數有多少個？

How many even numbers between 4000 and 7000 have four different digits?

Let the 4-digit number be $\overline{ABCD}$, where A can be 4, 5 or 6.

When $A = 4$, D has 4 choices (0, 2, 6, 8), B has 8 choices, C has 7 choices.

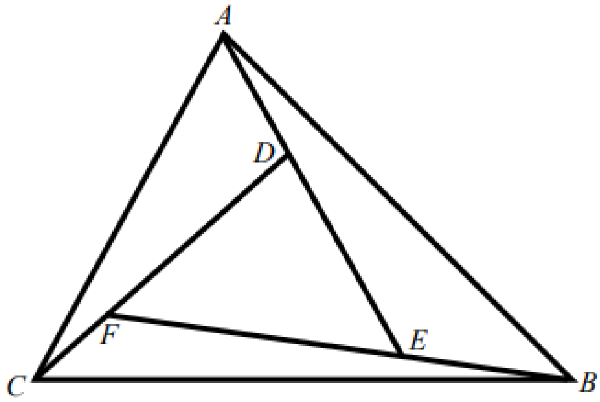
When $A = 5$, D has 5 choices (0, 2, 4, 6, 8), B has 8 choices, C has 7 choices.

When $A = 6$, D has 4 choices (0, 2, 4, 8), B has 8 choices, C has 7 choices.

Total numbers of different digit even numbers = 4×8×7 + 5×8×7 + 4×8×7 = 728

10. 在圖三中，BEF、ADE 及 CFD 是直線，使得 $BE : EF = 1 : 2$，$AD : DE = 1 : 3$ 及 $CF : FD = 1 : 4$。若 ΔDEF 的面積是 24 平方單位，求 ΔABC 的面積。

In Figure 3, BEF, ADE and CFD are straight lines such that $BE : EF = 1 : 2$, $AD : DE = 1 : 3$ and $CF : FD = 1 : 4$. If the area of ΔDEF is 24 square unit, find the area of ΔABC .



圖三 Figure 3

Reference: 1992 HG7

Let S denote the area of triangle. $S_{\Delta DEF} = 24$

$S_{\Delta ADF} = 24\times\frac{1}{3} = 8 \Rightarrow S_{\Delta AEF} = 8 + 24 = 32$

$S_{\Delta ABE} = 32\times\frac{1}{2} = 16 \Rightarrow S_{\Delta ABF} = 32 + 16 = 48$

$S_{\Delta ACF} = 8\times\frac{1}{4} = 2 \Rightarrow S_{\Delta ACD} = 8 + 2 = 10$

$S_{\Delta BDE} = 24\times\frac{1}{2} = 12 \Rightarrow S_{\Delta BDF} = 24 + 12 = 36$

$S_{\Delta BCF} = 36\times\frac{1}{4} = 9$

$S_{\Delta BDE} = 24 + 16 + 10 + 9 = 59$

11. 若 $\log_9 x^{18} = (\log_3 x)^3$，求 x 的最小值。

If $\log_9 x^{18} = (\log_3 x)^3$, find the least value of x .

$\frac{18\log_3 x}{\log_3 9} = (\log_3 x)^3 \Rightarrow \frac{18\log_3 x}{2} = (\log_3 x)^3 \Rightarrow 9\log_3 x = (\log_3 x)^3$

$\log_3 x = 0$ or $\log_3 x = 3$ or $\log_3 x = -3$

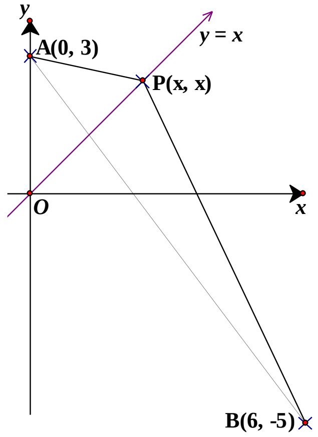
$x = 1$ or $x = 27$ or $x = \frac{1}{27}$

The least value of $x = \frac{1}{27}$.

12. 設 $f(x)=\sqrt{(x-3)^2+x^2}+\sqrt{(x-6)^2+(x+5)^2}$，其中 x 為一實數。 求 $f(x)$ 的最小值。

Let $f(x)=\sqrt{(x-3)^2+x^2}+\sqrt{(x-6)^2+(x+5)^2}$, where x is a real number.

Find the minimum value of $f(x)$. **(Reference: 2010 FG4.2, 2015 HI9)**



Consider the following problem:

Given the straight line $y=x$ and two fixed points $A(0, 3)$ and $B(6, -5)$ not on the straight line. $P(x, x)$ is any point on the straight line.

$f(x)=\sqrt{(x-3)^2+x^2}+\sqrt{(x-6)^2+(x+5)^2}$

$= AP + PB$

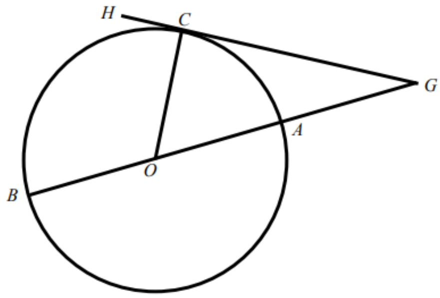
By triangle inequality, $AP + PB \geq AB$

$\therefore f(x)\geq\sqrt{(6-0)^2+(-5-3)^2}=10$

The minimum value of $f(x)$ is 10 .

13. 在圖四中，O 是圓的圓心。直徑 BA 延長至點 G 使得 GH 切圓於 C 點。若 $OA=5$ 及 $GC=12$，求 BC 的長度。

In Figure 4, O is the centre of the circle. The diameter BA is produced to a point G such that GH is a tangent to the circle at C. If $OA = 5$ and $GC = 12$, find the length of BC.



圖四 Figure 4

Let $GA = x$

$OC \perp CG$ (tangent $\perp$ radius)

In ΔOCG, $(x + 5)^2 = 5^2 + 12^2$ (Pythagoras' theorem)

$x=8\Rightarrow\cos\angle CGO=\frac{12}{13}$

In ΔBCG, $BC^2=18^2+12^2-2\times18\times12\times\frac{12}{13}=\frac{900}{13}$

$BC=\frac{30\sqrt{13}}{13}$

14. 對任意實數 x，函數 $f(x)$有以下性質 $f(x) + f(x-1) = x^2$。若 $f(19) = 94$，求 $f(94)$的值。

For each real number x, the function $f(x)$ has the following property $f(x) + f(x-1) = x^2$.

If $f(19) = 94$, find the value of $f(94)$.

$f(20) + f(19) = 20^2 \Rightarrow f(20) = 20^2 - 94$

$f(21) + f(20) = 21^2 \Rightarrow f(21) = 21^2 - f(20) = 21^2 - 20^2 + 94$

$f(22) + f(21) = 22^2 \Rightarrow f(22) = 22^2 - f(21) = 22^2 - 21^2 + 20^2 - 94$

Deductively, $f(94) = 94^2 - 93^2 + 92^2 - 91^2 + \cdots - 21^2 + 20^2 - 94$

$f(94) = (94 + 93)(94 - 93) + (92 + 91)(92 - 91) + \cdots + (22 + 21)(22 - 21) + 20^2 - 94$

$= 94 + 93 + \cdots + 22 + 21 + 400 - 94$

$=\frac{1}{2}(21+94)\cdot74+306$

$= 115 \times 37 + 306$

$= 4255 + 306 = 4561$

15. 已知 $(x + 2y)^2 = 2xy - 3x + 6y - 9$。若 x 及 y 為實數，求 $x + y$ 的值。

Given that $(x + 2y)^2 = 2xy - 3x + 6y - 9$. If x and y are real number, find the value of $x + y$.

$(x + 2y)^2 = 2xy - 3x + 6y - 9$

$x^2 + 4xy + 4y^2 = 2xy - 3x + 6y - 9$

$x^2 + (2y + 3)x + 4y^2 - 6y + 9 = 0$

$\Delta = (2y + 3)^2 - 4(4y^2 - 6y + 9)$

$= 4y^2 + 12y + 9 - 16y^2 + 24y - 36$

$= -12y^2 + 36y - 27$

$= -3(4y^2 - 12y + 9)$

$= -3(2y - 3)^2 \le 0$

For real values of x, $\Delta = 0$

$y = 1.5$

$x = -\dfrac{b}{2a} = -\dfrac{2\times 1.5+3}{2} = -3$

$x + y = -3 + 1.5 = -1.5$

Method 2

$(x + 2y)^2 = 2xy - 3x + 6y - 9$

$x^2 + 4xy + 4y^2 = 2xy - 3x + 6y - 9$

$(x^2 + 2xy + y^2) +(3x + 3y) + 3y^2 - 9y + 9 = 0$

$(x + y)^2 + 2\times 1.5(x + y) + 1.5^2 + 3y^2 - 9y + 6.75 = 0$

$(x + y + 1.5)^2 + 0.75(4y^2 - 12y + 9) = 0$

$(x + y + 1.5)^2 + 0.75(2y - 3)^2 = 0$

sum of two non-negative numbers = 0 $\Rightarrow$ each term = 0

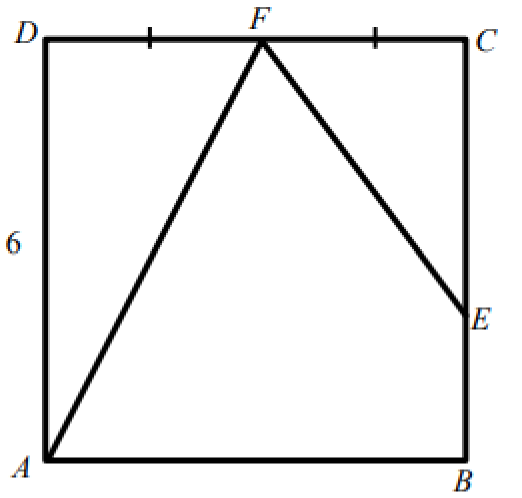
$x + y + 1.5 = 0$ and $y = 1.5$

$x = -3$

$x + y = -1.5$

Paper 2

1. 在圖一中，$ABCD$ 是一個邊長為 6 的正方形。F 是 CD 的中點。若 $\angle FAB = \angle AFE$，求 BE 的長度。
In Figure 1, $ABCD$ is a square of sides 6 units. F is the mid-point of CD. If $\angle FAB = \angle AFE$, find the length of BE.



圖一 Figure 1

Let $\angle FAB = \angle AFE = \theta$.
Then $\angle DAF = 90° - \theta$, $\angle AFD = 90° - (90° - \theta) = \theta$
$CF = FD = 3$

In ΔADF, $\tan\theta = \frac{6}{3} = 2$

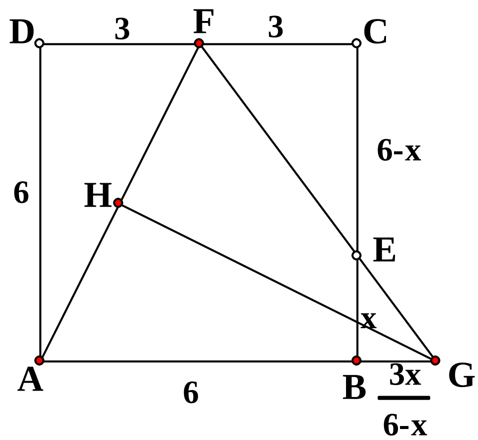
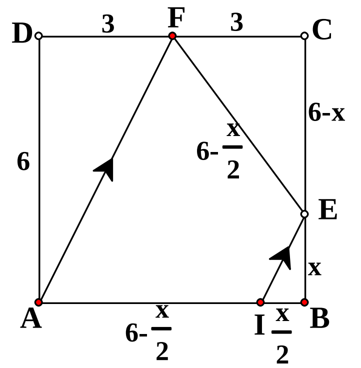
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\times 2}{1-2^2} = -\frac{4}{3}$$

$\angle CFE = 180° - \angle DFE = 180° - 2\theta$ (adj. $\angle$s on st. line)

In ΔCEF, $\tan\angle CFE = \frac{CE}{CF}$

$$CE = 3\tan(180° - 2\theta) = 3\times\left[-\left(-\frac{4}{3}\right)\right] = 4$$

$BE = 6 - 4 = 2$

Method 2 $CF = FD = 3$ $AF = \sqrt{45} = 3\sqrt{5}$ (Pythagoras' theorem) Let $BE = x$, then $CE = 6 - x$ It is easy to show that $\Delta CEF \sim \Delta BEG$ (A.A.A.) $BG : x = 3 : (6 - x)$ (cor. sides ~ Δs) $BG = \frac{3x}{6-x}$, $AG = AB + BG = 6 + \frac{3x}{6-x} = \frac{36-3x}{6-x}$ Let the mid-point of AF be H. $AH = \frac{3\sqrt{5}}{2}$ $\Delta AGH \cong \Delta FGH$ (S.A.S.) $\angle AHG = \angle FHG = 90°$ (cor. $\angle$s $\cong\Delta$s) $\angle GAH = \angle AFD$ (alt. $\angle$s DF // AB) $\Delta ADF \sim \Delta GHA$ (A.A.A.) $DF : AF = AH : AG$ (cor. sides, ~Δs) $\frac{3}{3\sqrt{5}} = \frac{3\sqrt{5}}{2\left(\frac{36-3x}{6-x}\right)}$ $24 - 2x = 5(6 - x) \Rightarrow BE = x = 2$	
Method 3 Draw EI // FA, cutting AB at I, then $AFEI$ is an isosceles trapezium. Let $BE = x$, $CE = 6 - x$, $CF = FD = 3$ $\angle BIE = \angle IAF$ (cor. $\angle$s, AF // IE) $= \angle AFD$ (alt. $\angle$s DF // AB) $\Delta ADF \sim \Delta EBI$ (A.A.A.) $BI : BE = DF : AD$ (cor. sides, ~Δs) $BI = \frac{x}{2} \Rightarrow AI = 6 - \frac{x}{2} = EF$ In ΔCEF, $3^2 + (6 - x)^2 = \left(6 - \frac{x}{2}\right)^2 \Rightarrow BE = x = 2$	

Method 4

Draw EI // FA, cutting AB at I, then $AFEI$ is an isosceles trapezium. Let $BE = x$, $CE = 6 - x$, $CF = FD = 3$

By the same arguments as in method 3, $AI = 6 - \frac{x}{2} = EF$

On EF, locate a point H so that $FH = 3$

$\Delta ADF \cong \Delta AHF$ (S.A.S.)

$AH = AD = 6$ (cor. sides, $\cong \Delta$s)

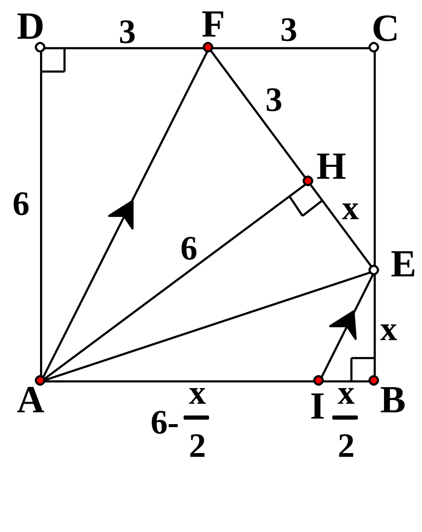
$\angle AHF = \angle ADF = 90°$ (cor. $\angle$s, $\cong \Delta$s)

$\Delta ABE \cong \Delta AHE$ (R.H.S.)

$\therefore HE = BE = x$ (cor. sides, $\cong \Delta$s)

$EF = FH + HE = 3 + x = 6 - \frac{x}{2}$

$BE = x = 2$



2. 設 $S = 2011^n + 2012^n + 2013^n + 2014^n + 2015^n + 2016^n + 2017^n + 2018^n + 2019^n$，其中 n 為一正整數。若 S 未能被 5 整除，求 S 的個位數。

Let $S = 2011^n + 2012^n + 2013^n + 2014^n + 2015^n + 2016^n + 2017^n + 2018^n + 2019^n$, where n is an integer. If S is not divisible by 5 , find the unit digit of S .

$2011^n \equiv 1 \pmod{10}$, $2015^n \equiv 5$, $2012^1 \equiv 2$, $2012^2 \equiv 4$, $2012^3 \equiv 8$, $2012^4 \equiv 6$, $2012^5 \equiv 2$, $\cdots$

$2013^1 \equiv 3$, $2013^2 \equiv 9$, $2013^3 \equiv 7$, $2013^4 \equiv 1$, $\cdots$

$2014^1 \equiv 4$, $2014^2 \equiv 6$, $2014^3 \equiv 4$, $\cdots$, $2016^n \equiv 6$,

$2017^1 \equiv 7$, $2017^2 \equiv 9$, $2017^3 \equiv 3$, $2017^4 \equiv 1$, $\cdots$

$2018^1 \equiv 8$, $2018^2 \equiv 4$, $2018^3 \equiv 2$, $2018^4 \equiv 6$, $\cdots$, $2019^1 \equiv 9$, $2019^2 \equiv 1$, $\cdots$

$\therefore$ When $n = 4k$, $S \equiv 1 + 6 + 1 + 6 + 5 + 6 + 1 + 6 + 1 \equiv 3$

When $n = 4k + 1$, $S \equiv 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 \equiv 45$ (which is divisible by 5)

When $n = 4k + 2$, $S \equiv 1 + 4 + 9 + 6 + 5 + 6 + 9 + 4 + 1 \equiv 45$ (which is divisible by 5)

When $n = 4k + 3$, $S \equiv 1 + 8 + 7 + 4 + 5 + 6 + 3 + 2 + 1 \equiv 45$ (which is divisible by 5)

$\because S$ is not divisible by 5

$\therefore S = 3$

3. 在圖二中，四個半徑分別為 8、5、5 及 r 的圓互相外切。求 r 的值。

In Figure 2, four circles of radii 8, 5, 5 and r are touching each other externally. Find the value of r .

Let E be the mid-point of BC.

$AE = 12$, $AB = AC = 13$, $BE = EC = 5$

$\Delta ABE \cong \Delta ACE$ (S.S.S.)

$AE \perp BC$ (cor. $\angle$s, $\cong\Delta$s)

$AD = 8 + r$, $DE = 12 - (8 + r) = 4 - r$

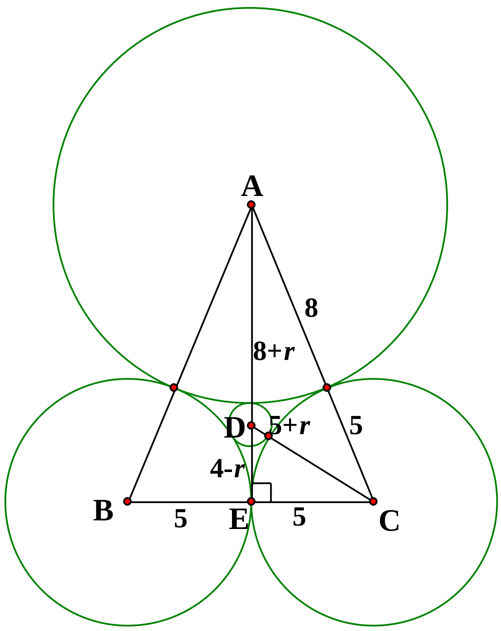
$CD = 5 + r$

$5^2 + (4 - r)^2 = (5 + r)^2$ (Pythagoras' theorem)

$25 + 16 - 8r + r^2 = 25 + 10r + r^2$

$18r = 16$

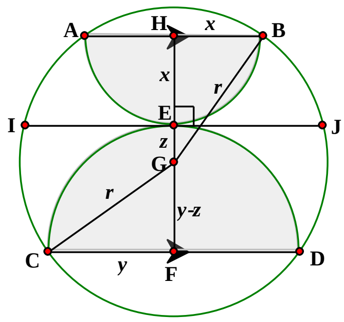
$r = \frac{8}{9}$



圖二 Figure 2

4. 已知 a、b、c、d 及 e 是連續正整數，其中 $a < b < c < d < e$。
若 $a + b + c + d + e$ 是一個立方數及 $b + c + d$ 是一個平方數，求 c 的最小可能值。
Given that a , b , c , d and e are consecutive positive integers, where $a < b < c < d < e$.
If $a + b + c + d + e$ is a perfect cube and $b + c + d$ is a perfect square, find the smallest possible value of c . (**Reference: 1998 HG4, 1999 FG3.1**)
$a = c - 2$, $b = c - 1$, c, $d = c + 1$, $e = c + 2$
$a + b + c + d + e = 5c = m^3$
$b + c + d = 3c = n^2$
The smallest value of $c = 3\times5^2 = 75$

5. $ABCD$ 是圓形而 ABE 及 CED 為半圓形互切於 E 在圓內。已知圓面積為 1 cm^2 及 $AB \,//\, CD$，求半圓形 ABE 及 CED 的面積之和。
$ABCD$ is a circle while ABE and CED are semi-circles touching each other at E inside the circle. Given the area of circle is 1 cm^2 and $AB \,//\, CD$, find the sum of the area of the semi-circles ABE and CED .



Let H be the mid-point of AB and G be the centre of the circle.
Join HG and produce it to cut CD at F.
$HG \perp AB$ (line joining centre to mid-point of chord $\perp$ chord)
$\angle AHG + \angle CFG = 180°$ (int. $\angle$s, $AB \,//\, CD$)
$\angle CFG = 180° - 90° = 90°$
$CF = FD$ (line joining centre $\perp$ to chord bisects chord)
Draw a common tangent IEJ through E, touching the two semi-circles at E and cuts the circle at I and J respectively.
H and F are the centres of the upper and the lower semi-circles. Join FE and HE .
$EH \perp IJ$, $EF \perp IJ$ (tangent $\perp$ radius)
$\angle HEJ + \angle FEJ = 180°$
$\therefore$ H, E, F are collinear
$HEF \perp AB$ (and CD) and must pass through the centre G.
Let $HB = HE = x$, $FC = FE = y$, $GC = GB = r$, then $HG = x + z$, $FG = y - z$
$(x + z)^2 + x^2 = r^2 \cdots\cdots (1)$ (Pythagoras' theorem on ΔBGH)
$(y - z)^2 + y^2 = r^2 \cdots\cdots (2)$ (Pythagoras' theorem on ΔCFG)
(1) = (2): $2x^2 + 2xz + z^2 = 2y^2 - 2yz + z^2$
$2(x^2 - y^2) + 2z(x + y) = 0$
$(x + y)(x - y + z) = 0$
$y = x + z \cdots\cdots (3)$
$\pi r^2 = 1 \cdots\cdots (4)$ ($\because$ area of the circle = 1 cm^2)
Sum of areas of the two semi-circles

$=\frac{\pi}{2}\left(x^2+y^2\right)$

$=\frac{\pi}{2}\left[x^2+(x+z)^2\right]$ (By (3))

$=\frac{\pi}{2}r^2$ (By (1))

$=\frac{1}{2}\text{ cm}^2$ (By (4))

6. 如果 $d=\log_2\left(\sqrt{2^2+2^{1013}+2^{2022}}-2\right)$，求 d 的值。

If $d=\log_2\left(\sqrt{2^2+2^{1013}+2^{2022}}-2\right)$, find the value of d.

$d=\log_2\left(\sqrt{2^2+2^{1013}+2^{2022}}-2\right)$

$=\log_2\left(\sqrt{2^2+2\times2\times2^{1011}+\left(2^{1011}\right)^2}-2\right)$

$=\log_2\left(\sqrt{\left(2+2^{1011}\right)^2}-2\right)$

$=\log_2\left(2+2^{1011}-2\right)$

$=\log_2\left(2^{1011}\right)$

$= 1011$

7. 求 $\sqrt{10000\times10002\times10004\times10006+16}$ 的值。

Find the value of $\sqrt{10000\times10002\times10004\times10006+16}$.

Let $x = 10003$, then $x - 3 = 10000$, $x - 1 = 10002$, $x + 1 = 10004$, $x + 3 = 10006$

$\sqrt{10000\times10002\times10004\times10006+16}$

$=\sqrt{(x-3)(x-1)(x+1)(x+3)+16}$

$=\sqrt{\left(x^2-9\right)\left(x^2-1\right)+16}$

$=\sqrt{x^4-10x^2+25}$

$=\sqrt{\left(x^2-5\right)^2}$

$= x^2 - 5 = (10000 + 3)^2 - 5$

$= 100000000 + 60000 + 9 - 5$

$= 100060004$