# Hong Kong Mathematics Olympiad (1984 – 1985)
## Sample Event (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 某兩數之和為 40，其積為 20。若該兩數倒數之和為 $a$，求 $a$ 的值。
The sum of two numbers is 40, and their product is 20.
If the sum of their reciprocals is $a$, find the value of $a$.

$a =$

(ii) 若一邊長 $(a + 1)$ cm 之正方體之總表面積為 $b$ cm$^2$，求 $b$ 的值。
If $b$ cm$^2$ is the total surface area of a cube of side $(a + 1)$ cm, find the value of $b$.

$b =$

(iii) 一袋內有 $b - 4$ 個白球，$b + 46$ 個紅球。若隨意於袋內取一球，而該球為白色之概率為 $\frac{c}{6}$，求 $c$ 的值。
One ball is taken at random from a bag containing $b - 4$ white balls and $b + 46$ red balls.
If $\frac{c}{6}$ is the probability that the ball is white, find the value of $c$.

$c =$

(iv) 若一邊長 $c$ cm 之正三角形之面積為 $d\sqrt{3}$ cm$^2$，求 $d$ 的值。
The length of a side of an equilateral triangle is $c$ cm. If its area is $d\sqrt{3}$ cm$^2$, find the value of $d$.

$d =$

**FOR OFFICIAL USE**

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# Hong Kong Mathematics Olympiad (1984 – 1985)
## Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $a = \log_5 \dfrac{(125)(625)}{25}$，求 $a$ 的值。

Find the value of $a$ if $a = \log_5 \dfrac{(125)(625)}{25}$.

$a =$

(ii) 若 $\left(r+\dfrac{1}{r}\right)^2 = a-2$ 且 $r^3+\dfrac{1}{r^3}=b$，求 $b$ 的值。

If $\left(r+\dfrac{1}{r}\right)^2 = a-2$ and $r^3+\dfrac{1}{r^3}=b$, find the value of $b$.

$b =$

(iii) 若 2 為方程 $x^3 + cx + 10 = b$ 之一根，求 $c$ 的值。
If one root of the equation $x^3 + cx + 10 = b$ is 2, find the value of $c$.

$c =$

(iv) 若 $9^{d+2} = (6489 + c) + 9^d$，求 $d$ 的值。
Find the value of $d$ if $9^{d+2} = (6489 + c) + 9^d$.

$d =$

**FOR OFFICIAL USE**

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Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 在以下數列中，求 $a$ 的值：

$1, 8, 27, 64, a, 216, \cdots\cdots$

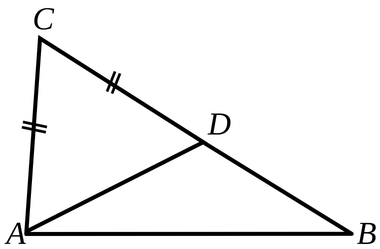
Find $a$ in the following sequence:

$1, 8, 27, 64, a, 216, \cdots\cdots$

$a =$

(ii) 在圖一中，$AC = CD$，$\angle CAB - \angle ABC = (a - 95)°$。若$\angle BAD = b°$，求 $b$ 的值。
In Figure 1, $AC = CD$ and $\angle CAB - \angle ABC = (a - 95)°$.
If $\angle BAD = b°$, find the value of $b$.

$b =$



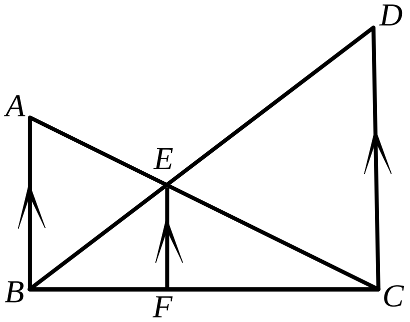

圖一 Figure 1

(iii) 一直綫過$(-1, 1)$及$(3, b - 6)$。若其 $y$ 截距為 $c$，求 $c$ 的值。
A line passes through the points $(-1, 1)$ and $(3, b - 6)$. If the $y$-intercept of the line is $c$, find the value of $c$.

$c =$

(iv) 在圖二中，$AB = c + 17$，$BC = 100$，$CD = 80$。若 $EF = d$，求 $d$ 的值。
In Figure 2, $AB = c + 17$, $BC = 100$, $CD = 80$. If $EF = d$, find the value of $d$.

$d =$




圖二 Figure 2

**FOR OFFICIAL USE**

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**Hong Kong Mathematics Olympiad (1984 – 1985)**
**Final Event 3 (Individual)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 在二時十五分，時鐘兩針所構成之銳角為 $\left(18\frac{1}{2}+a\right)^{\circ}$，求 $a$ 的值。

The acute angle formed by the hands of a clock at 2:15 is $\left(18\frac{1}{2}+a\right)^{\circ}$.

Find the value of $a$.

$a =$

(ii) 若 $(x+y)^a$ 的展開式之係數總和是 $b$，求 $b$ 的值。
If the sum of the coefficients in the expansion of $(x+y)^a$ is $b$, find the value of $b$.

$b =$

(iii) 若 $\mathrm{f}(x) = x - 2$，$F(x, y) = y^2 + x$，且 $c = F(3, \mathrm{f}(b))$，求 $c$ 的值。
If $\mathrm{f}(x) = x - 2$, $F(x, y) = y^2 + x$ and $c = F(3, \mathrm{f}(b))$, find the value of $c$.

$c =$

(iv) $x$，$y$ 為實數。若 $x + y = c - 195$ 及 $d$ 為 $xy$ 之最大值，求 $d$ 的值。
$x$, $y$ are real numbers. If $x + y = c - 195$ and $d$ is the maximum value of $xy$, find the value of $d$.

$d =$

**FOR OFFICIAL USE**

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**Hong Kong Mathematics Olympiad (1984 – 1985)**
**Final Event 4 (Individual)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若兩綫 $x + 2y + 3 = 0$ 及 $4x - ay + 5 = 0$ 互相垂直，求 $a$ 的值。
If the lines $x + 2y + 3 = 0$ and $4x - ay + 5 = 0$ are perpendicular to each other, find the value of $a$.

$a =$

(ii) 在圖一中，$ABCD$ 為一梯形，$AB$ 與 $DC$ 平行且$\angle ABC = \angle DCB = 90°$。若 $AB = a$，$BC = CD = 8$ 及 $AD = b$，求 $b$ 的值。
In Figure 1, $ABCD$ is a trapezium with $AB$ parallel to $DC$ and $\angle ABC = \angle DCB = 90°$. If $AB = a$, $BC = CD = 8$ and $AD = b$, find the value of $b$.

$b =$

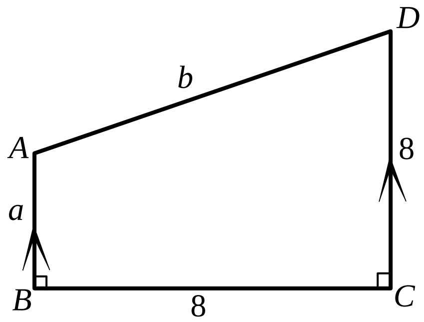


Figure 1
圖一

(iii) 在圖二中，$BD = \frac{b}{2}$，$DE = 4$，$EC = 3$。若$\Delta AEC$ 之面積為 24 及$\Delta ABC$ 之面積為 $c$，求 $c$ 的值。
In Figure 2, $BD = \frac{b}{2}$, $DE = 4$, $EC = 3$.
If the area of $\Delta AEC$ is 24 and the area of $\Delta ABC$ is $c$, find the value of $c$.

$c =$

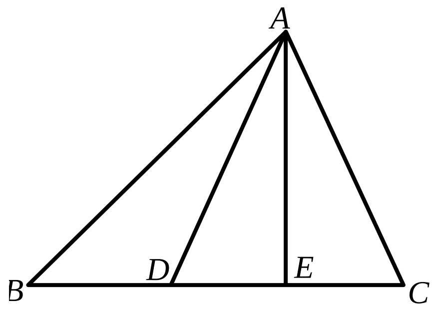


Figure 2
圖二

(iv) 若 $3x^3 - 2x^2 + dx - c$ 可被 $x - 1$ 整除，求 $d$ 的值。
If $3x^3 - 2x^2 + dx - c$ is divisible by $x - 1$, find the value of $d$.

$d =$

**FOR OFFICIAL USE**

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**Hong Kong Mathematics Olympiad (1984 – 1985)**
**Final Event 5 (Individual)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $1+2+3+4+\ldots+t=36$，求 $t$ 的值。
If $1+2+3+4+\ldots+t=36$, find the value of $t$.

$t=$

(ii) 若 $\sin u° = \frac{2}{\sqrt{t}}$ 且 $90 < u < 180$，求 $u$ 的值。
If $\sin u° = \frac{2}{\sqrt{t}}$ and $90 < u < 180$, find the value of $u$.

$u=$

(iii) 在圖一中，$\angle ABC = 30°$，且 $AC = (u-90)$ cm。若$\Delta ABC$之外接圓半徑為 $v$ cm，求 $v$ 的值。
In Figure 1, $\angle ABC = 30°$ and $AC = (u-90)$ cm.
If the radius of the circumcircle of $\Delta ABC$ is $v$ cm, find the value of $v$.

$v=$

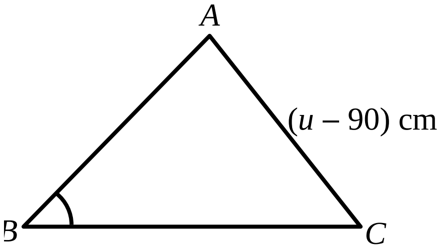


Figure 1
圖一

(iv) 在圖二中，$\Delta PAB$ 由切於圓之三切綫形成，且 $O$ 為圓心，
若 $\angle APB = (v-5)°$，且 $\angle AOB = w°$,，求 $w$ 的值。
In Figure 2, $\Delta PAB$ is formed by the 3 tangents of the circle with centre $O$.
If $\angle APB = (v-5)°$ and $\angle AOB = w°$, find the value of $w$.

$w=$

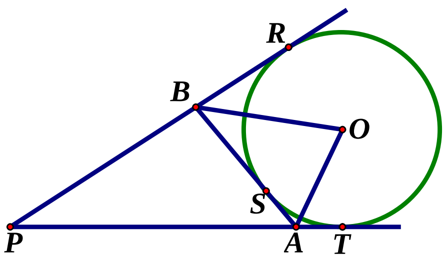


Figure 2
圖二

**FOR OFFICIAL USE**

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# Hong Kong Mathematics Olympiad (1984 – 1985)
## Sample Event (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $a*b = ab + 1$，且 $s = (2*4)*2$，求 $s$ 的值。
If $a*b = ab + 1$ and $s = (2*4)*2$, find the value of $s$.

$s =$

(ii) 若第 $n$ 個質數為 $s$，求 $n$ 的值。
If the $n^{\text{th}}$ prime number is $s$, find the value of $n$.

$n =$

(iii) 若 $K = \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\left(1-\frac{1}{50}\right)$，試以最簡單之分數表 $K$。
If $K = \left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\left(1-\frac{1}{50}\right)$,
find the value of $K$ in the simplest fractional form.

$K =$

(iv) 一正方形內接於一個半徑為 10 之圓。若正方形之面積為 $A$，求 $A$ 的值。
If $A$ is the area of a square inscribed in a circle of radius 10, find the value of $A$.

$A =$

**FOR OFFICIAL USE**

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## Hong Kong Mathematics Olympiad (1984 – 1985)
## Final Event 6 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) $p$、$q$、$r$ 之平均數為 4。 $p$、$q$、$r$、$x$ 之平均數為 5。求 $x$ 的值。
The average of $p$, $q$, $r$ is 4 . The average of $p$, $q$, $r$, $x$ is 5. Find the value of $x$ .

$x =$

(ii) 一行車速率為 60 km/h 的貨車之一輪每秒轉動 4 周，
若其直徑為 $\frac{y}{6\pi}$ m，求 $y$ 的值。
A wheel of a truck travelling at 60 km/h makes 4 revolutions per second.
If its diameter is $\frac{y}{6\pi}$ m, find the value of $y$ .

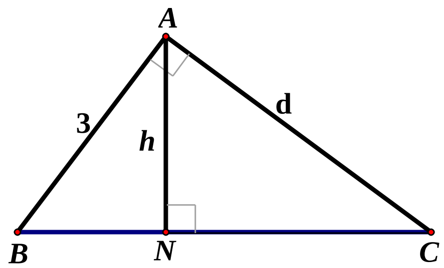
$y =$

(iii) If $\sin(55 - y)^\circ = \frac{d}{x}$, find the value of $d$ .
若 $\sin(55 - y)^\circ = \frac{d}{x}$，求 $d$ 的值。

$d =$

(iv) 如附圖所示，$BA \perp AC$ 及 $AN \perp BC$。若 $AB = 3$，$AC = d$，$AN = h$，求 $h$ 的值。
In the figure, $BA \perp AC$ and $AN \perp BC$. If $AB = 3$, $AC = d$, $AN = h$,
find the value of $h$ .

$h =$




**FOR OFFICIAL USE**

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**Hong Kong Mathematics Olympiad (1984 – 1985)**
**Final Event 7 (Group)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 設 $M=\dfrac{78^3+22^3}{78^2-78\times 22+22^2}$。求 $M$ 的值。

Let $M=\dfrac{78^3+22^3}{78^2-78\times 22+22^2}$. Find the value of $M$.

$M=$

(ii) 正整數 $N$ 分別被 6、5、4、3 及 2 除時，其餘數依次為 5、4、3、2 及 1。求 $N$ 之最小值。

When the positive integer $N$ is divided by 6, 5, 4, 3 and 2, the remainders are 5, 4, 3, 2 and 1 respectively. Find the least value of $N$.

$N=$

(iii) 一人以 4 km/h 之速率步行 10 km，再以 6 km/h 之速率步行另 10 km。若全程之平均速率為 $x$ km/h，求 $x$ 的值。

A man travels 10 km at a speed of 4 km/h and another 10 km at a speed of 6 km/h. If the average speed of the whole journey is $x$ km/h, find the value of $x$.

$x=$

(iv) 若 $S = 1 + 2 - 3 - 4 + 5 + 6 - 7 - 8 + \ldots + 1985$，求 $S$ 的值。

If $S = 1 + 2 - 3 - 4 + 5 + 6 - 7 - 8 + \ldots + 1985$, find the value of $S$.

$S=$

**FOR OFFICIAL USE**

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**Final Event 8 (Group)**

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

$M$、$N$ 均為小於 10 之正整數，且 $258024M8 \times 9 = 2111110N \times 11$。

$M$, $N$ are positive integers less than 10 and $258024M8 \times 9 = 2111110N \times 11$.

(i) 求 $M$ 的值。
Find the value of $M$.

$M =$

(ii) 求 $N$ 的值。
Find the value of $N$.

$N =$

(iii) 一凸 20 邊形有 $x$ 條對角線。求 $x$ 的值。
A convex 20-sided polygon has $x$ diagonals. Find the value of $x$.

$x =$

(iv) 若 $y = ab + a + b + 1$ 且 $a = 99$，$b = 49$，求 $y$ 的值。
If $y = ab + a + b + 1$ and $a = 99$, $b = 49$, find the value of $y$.

$y =$

**FOR OFFICIAL USE**

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# Final Event 9 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) $\Delta LMN$ 之三邊長分別為 8、15 及 17。若 $\Delta LMN$ 之面積為 $A$，求 $A$ 的值。
The lengths of the 3 sides of $\Delta LMN$ are 8, 15 and 17 respectively.
If the area of $\Delta LMN$ is $A$, find the value of $A$.

$A$ =

(ii) 若 $\Delta LMN$ 之內接圓之半徑為 $r$，求 $r$ 的值。
If $r$ is the length of the radius of the circle inscribed in $\Delta LMN$, find the value of $r$.

$r$ =

(iii) 若某年五月第 $r$ 日為星期五，且同年五月第 $n$ 日為星期一，其中 $15 < n < 25$，求 $n$ 的值。
If the $r^{\text{th}}$ day of May in a year is Friday and the $n^{\text{th}}$ day of May in the same year is Monday, where $15 < n < 25$, find the value of $n$.

$n$ =

(iv) 若一凸 $n$ 邊形之內角和為 $x°$，求 $x$ 的值。
If the sum of the interior angles of an $n$-sided convex polygon is $x°$, find the value of $x$.

$x$ =

**FOR OFFICIAL USE**

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# Hong Kong Mathematics Olympiad (1984 – 1985)
## Final Event 10 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 三連續奇數(最小者為 $k$)之和為 51。求 $k$ 的值。
The sum of 3 consecutive odd integers (the smallest being $k$) is 51.
Find the value of $k$.

$k =$

(ii) 若 $x^2 + 6x + k \equiv (x + a)^2 + C$，且 $a$、$C$ 為常數，求 $C$ 的值。
If $x^2 + 6x + k \equiv (x + a)^2 + C$, where $a$, $C$ are constants, find the value of $C$.

$C =$

(iii) 若 $\frac{p}{q} = \frac{q}{r} = \frac{r}{s} = 2$ 且 $R = \frac{p}{s}$，求 $R$ 的值。
If $\frac{p}{q} = \frac{q}{r} = \frac{r}{s} = 2$ and $R = \frac{p}{s}$, find the value of $R$.

$R =$

(iv) 若 $A = \frac{3^n \cdot 9^{n+1}}{27^{n-1}}$，求 $A$ 的值。
If $A = \frac{3^n \cdot 9^{n+1}}{27^{n-1}}$, find the value of $A$.

$A =$

**FOR OFFICIAL USE**

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