Hong Kong Mathematics Olympiad (1986 – 1987)
Sample Event (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $x^2 - 8x + 26 \equiv (x + k)^2 + a$，求 a 的值。
If $x^2 - 8x + 26 \equiv (x + k)^2 + a$, find the value of a.

$a =$

(ii) 若 $\sin a° = \cos b°$，其中 $270 < b < 360$，求 b 的值。
If $\sin a° = \cos b°$, where $270 < b < 360$, find the value of b.

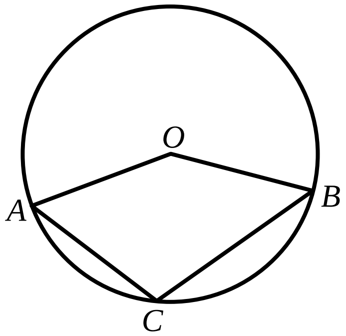
$b =$

(iii) X 以\$$b$ 出售一貨品與 Y 而虧蝕 30%。若 X 購入該貨品之成本為\$$c$，求 c 的值。
X sold an article to Y for \$$b$ at a loss of 30%.
If the cost price of the article for X is \$$c$, find the value of c.

$c =$

(iv) 附圖中，O 為圓心。若 $\angle ACB = \frac{3c°}{10}$ 及 $\angle AOB = d°$，求 d 的值。
In the figure, O is the centre of the circle.
If $\angle ACB = \frac{3c°}{10}$ and $\angle AOB = d°$, find the value of d.

$d =$



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Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1986 – 1987)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $A = 11 + 12 + 13 + \ldots + 29$，求 A 的值。
If $A = 11 + 12 + 13 + \ldots + 29$, find the value of A.

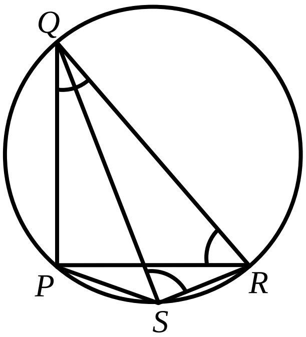
$A =$

(ii) 若 $\sin A° = \cos B°$，其中 $0 < B < 90$，求 B 的值。
If $\sin A° = \cos B°$, where $0 < B < 90$, find the value of B.

$B =$

(iii) 附圖中，$\angle PQR = B°$，$\angle PRQ = 50°$。若 $\angle QSR = n°$，求 n 的值。
In the given figure, $\angle PQR = B°$, $\angle PRQ = 50°$. If $\angle QSR = n°$, find the value of n.

$n =$



(iv) 由 1 至 n 號卡片中隨意抽出一張。若得到 5 之倍數之概率為 $\frac{1}{m}$，求 m 的值。
n cards are marked from 1 to n and one is drawn at random. If the chance of it being a multiple of 5 is $\frac{1}{m}$, find the value of m.

$m =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1986 – 1987)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 某球體之半徑為 r，體積為 36π，求 r 的值。
The volume of a sphere with radius r is 36π, find the value of r.

$r =$

(ii) 若 $r^x + r^{1-x} = 4$，且 $x > 0$，求 x 的值。
If $r^x + r^{1-x} = 4$ and $x > 0$, find the value of x.

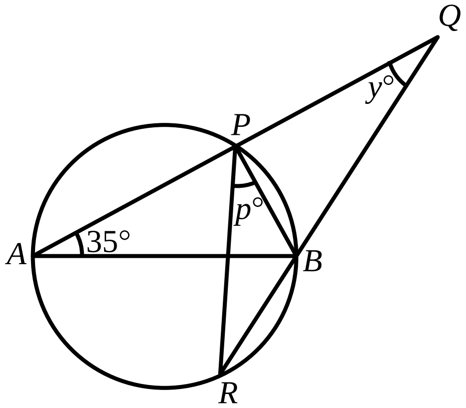
$x =$

(iii) 若 $a : b = 5 : 4$，$b : c = 3 : x$ 且 $a : c = y : 4$，求 y 的值。
In $a : b = 5 : 4$, $b : c = 3 : x$ and $a : c = y : 4$, find the value of y.

$y =$

(iv) 附圖中，AB 為該圓之直徑。APQ 及 RBQ 為直綫。若 $\angle PAB = 35°$，$\angle PQB = y°$ 及 $\angle RPB = p°$，求 p 的值。
In the figure, AB is a diameter of the circle. APQ and RBQ are straight lines.
If $\angle PAB = 35°$, $\angle PQB = y°$ and $\angle RPB = p°$, find the value of p.

$p =$



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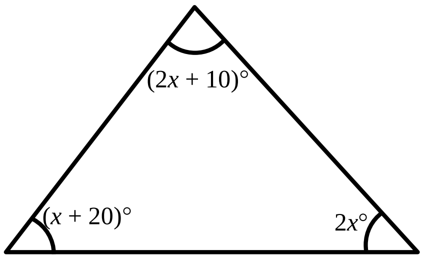
Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1986 – 1987)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 如圖所示，求 x 的值。
In the figure, find the value of x.



$x =$

(ii) P，Q 之坐標依次為$(a, 2)$及$(x, -6)$。若 PQ 的中點之坐標為$(18, b)$，求 a 的值。
The coordinates of the points P and Q are $(a, 2)$ and $(x, -6)$ respectively.
If the coordinates of the mid-point of PQ is $(18, b)$, find the value of a.

$a =$

(iii) 某人以均勻速度 a km/h 由 X 往 Y，並以均勻速度 $2a$ km/h 由 Y 返 X。
若其平均速度為 c km/h，求 c 的值。
A man travels from X to Y at a uniform speed of a km/h and returns at a uniform speed of $2a$ km/h. If his average speed is c km/h, find the value of c.

$c =$

(iv) 若 $f(y) = 2y^2 + cy - 1$，求 $f(4)$ 的值。
If $f(y) = 2y^2 + cy - 1$, find the value of $f(4)$.

$f(4)=$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1986 – 1987)
Final Event 4 (Individual)

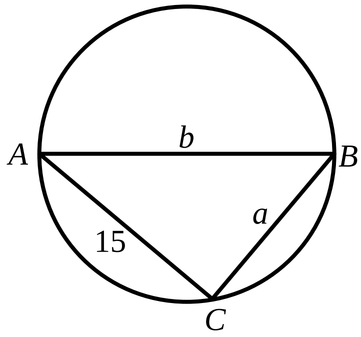
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若曲線 $y = 2x^2 - 8x + a$ 與 x-軸相切，求 a 的值。
If the curve $y = 2x^2 - 8x + a$ touches the x-axis, find the value of a.

$a =$

(ii) 附圖中，AB 為該圓之直徑。若 $AC = 15$，$BC = a$ 及 $AB = b$，求 b 的值。
In the figure, AB is a diameter of the circle. If $AC = 15$, $BC = a$ and $AB = b$, find the value of b.

$b =$



(iii) 直綫 $5x + by + 2 = d$ 過點(40, 5)。求 d 的值。
The line $5x + by + 2 = d$ passes through (40, 5). Find the value of d.

$d =$

(iv) X 以\$$d$ 出售一貨品與 Y，得利潤 2.5%。若 X 購入該貨品之成本為\$$K$，求 K 的值。
X sold an article to Y for \$$d$ at a profit of 2.5%. If the cost price of the article for X is \$$K$, find the value of K.

$K =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
			+	Bonus score			Time		
			Total score					Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 設 $x = 19.\dot{8}\dot{7}$。若 $19.\dot{8}\dot{7} = \frac{a}{99}$，求 a 的值。

(提示：$99x = 100x - x$)

Let $x = 19.\dot{8}\dot{7}$. If $19.\dot{8}\dot{7} = \frac{a}{99}$, find the value of a.

(Hint: $99x = 100x - x$)

$a =$

(ii) 若 $f(y) = 4\sin y°$，且 $f(a - 18) = b$，求 b 的值。

If $f(y) = 4\sin y°$ and $f(a - 18) = b$, find the value of b.

$b =$

(iii) 若 $\frac{\sqrt{3}}{b\sqrt{7}-\sqrt{3}} = \frac{2\sqrt{21}+3}{c}$，求 c 的值。

If $\frac{\sqrt{3}}{b\sqrt{7}-\sqrt{3}} = \frac{2\sqrt{21}+3}{c}$, find the value of c.

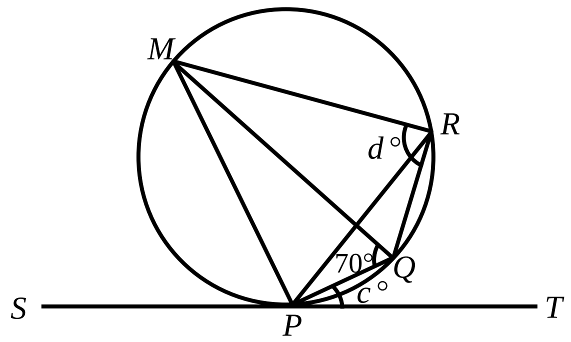
$c =$

(iv) 附圖中，ST 與圓相切於 P。若 $\angle MQP = 70°$，$\angle QPT = c°$ 及 $\angle MRQ = d°$，求 d 的值。

In the figure, ST is a tangent to the circle at P.

If $\angle MQP = 70°$, $\angle QPT = c°$ and $\angle MRQ = d°$, find the value of d.

$d =$



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Score for accuracy × Mult. factor for speed =

+ Bonus score

Total score

Team No.

Time

Min. Sec.

Hong Kong Mathematics Olympiad (1986 – 1987)
Sample Event (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $100A = 35^2 - 15^2$，求 A 的值。
If $100A = 35^2 - 15^2$, find the value of A.

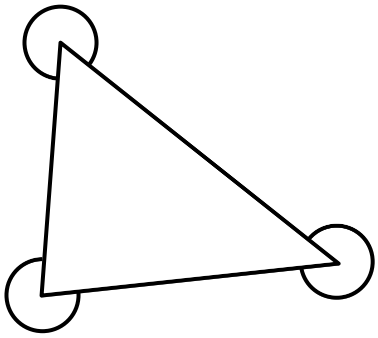
$A =$

(ii) 若 $(A - 1)^6 = 27^B$，求 B 的值。
If $(A - 1)^6 = 27^B$, find the value of B.

$B =$

(iii) 附圖所示三角之和是 $C°$。求 C 的值。
In the given diagram, the sum of the three marked angles is $C°$. Find the value of C.

$C =$



(iv) 若直綫 $x + 2y + 1 = 0$ 及 $9x + Dy + 1 = 0$ 互相平行，求 D 的值。
If the lines $x + 2y + 1 = 0$ and $9x + Dy + 1 = 0$ are parallel, find D.

$D =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1986 – 1987)
Final Event 6 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 α、β 為 $x^2 - 10x + 20 = 0$ 之根，且 $p = \alpha^2 + \beta^2$，求 p 的值。
If α, β are the roots of $x^2 - 10x + 20 = 0$, and $p = \alpha^2 + \beta^2$, find the value of p.

$p =$

(ii) 一正三角形之周界為 p。若其面積為 $k\sqrt{3}$，求 k 的值。
The perimeter of an equilateral triangle is p. If its area is $k\sqrt{3}$, find the value of k.

$k =$

(iii) 一正 N 邊形之每一內角為 140°。求 N 的值。
Each interior angle of an N-sided regular polygon is 140°. Find the value of N.

$N =$

(iv) 若 $M = (10^2 + 10\times1 + 1^2)(10^2 - 1^2)(10^2 - 10\times1 + 1^2)$，求 M 的值。
If $M = (10^2 + 10\times1 + 1^2)(10^2 - 1^2)(10^2 - 10\times1 + 1^2)$, find the value of M.

$M =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1986 – 1987)
Final Event 7 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 在下午三點三十分時，時鐘兩針所構成之銳角為 $A°$。求 A 的值。
The acute angle formed by the hands of a clock at 3:30 p.m. is $A°$. Find the value of A.

$A =$

(ii) 若 $\tan(3A + 15)° = \sqrt{B}$，求 B 的值。
If $\tan(3A + 15)° = \sqrt{B}$, find the value of B.

$B =$

(iii) 若 $\log_{10}AB = C \log_{10}15$，求 C 的值。
If $\log_{10}AB = C \log_{10}15$, find the value of C.

$C =$

(iv) 點 $(1, 3)$、$(4, 9)$ 及 $(2, D)$ 共线。求 D 的值。
The points $(1, 3)$, $(4, 9)$ and $(2, D)$ are collinear. Find the value of D.

$D =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1986 – 1987)
Final Event 8 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $A=\dfrac{5\sin\theta+4\cos\theta}{3\sin\theta+\cos\theta}$，且 $\tan\theta=2$，求 A 的值。

If $A=\dfrac{5\sin\theta+4\cos\theta}{3\sin\theta+\cos\theta}$ and $\tan\theta=2$, find the value of A.

$A=$

(ii) 若 $x+\dfrac{1}{x}=2A$，且 $x^3+\dfrac{1}{x^3}=B$，求 B 的值。

If $x+\dfrac{1}{x}=2A$, and $x^3+\dfrac{1}{x^3}=B$, find the value of B.

$B=$

(iii) 共有 N 個 α 值可滿足方程 $\cos^3\alpha-\cos\alpha=0$，其中 $0°\le\alpha\le 360°$。求 N 的值。

There are exactly N values of α satisfying the equation $\cos^3\alpha-\cos\alpha=0$, where $0°\le\alpha\le 360°$. Find the value of N.

$N=$

(iv) 若某年五月第 N 日為星期四，且同年五月第 K 日為星期一，其中 $10<K<20$，求 K 的值。

If the N^{th} day of May in a year is Thursday and the K^{th} day of May in the same year is Monday, where $10<K<20$, find the value of K.

$K=$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1986 – 1987)
Final Event 9 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

在所示乘法中，不同字母代表由 0 至 9 之不同整數。

In the given multiplication, different letters represent different integers ranging from 0 to 9.

$$\begin{array}{rcccc} & A & B & C & D \\ \times & & & & 9 \\ \hline & D & C & B & A \end{array}$$

(i) 求 A 的值。
Find the value of A .

$A =$

(ii) 求 B 的值。
Find the value of B .

$B =$

(iii) 求 C 的值。
Find the value of C .

$C =$

(iv) 求 D 的值。
Find the value of D .

$D =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+	Bonus score			Time	
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1986 – 1987)
Final Event 10 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) p、q、r 及 s 之平均數為 5。
p、q、r、s 及 A 之平均數為 8。求 A 的值。
The average of p, q, r and s is 5.
The average of p, q, r, s and A is 8. Find the value of A.

$A =$

(ii) 若直綫 $3x - 2y + 1 = 0$ 及 $Ax + By + 1 = 0$ 互相垂直，求 B 的值。
If the lines $3x - 2y + 1 = 0$ and $Ax + By + 1 = 0$ are perpendicular, find the value of B.

$B =$

(iii) 若 $Cx^3 - 3x^2 + x - 1$ 除以 $x + 1$ 得之餘數為 -7。求 C 的值。
When $Cx^3 - 3x^2 + x - 1$ is divided by $x + 1$, the remainder is -7. Find the value of C.

$C =$

(iv) 若 P、Q 為正整數使 $P + Q + PQ = 90$，且 $D = P + Q$，求 D 的值。
(提示：因式分解 $1 + P + Q + PQ$)
If P, Q are positive integers such that $P + Q + PQ = 90$ and $D = P + Q$, find the value of D. (Hint: Factorise $1 + P + Q + PQ$)

$D =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
			+ Bonus score				Time		
			Total score					Min.	Sec.