

第六屆香港數學競賽 (1988/89)

決賽規則

1. 競賽共分十項，個人及團體各佔五項。
2. 每隊由已報名參加初賽的同學組成。其中任何四位可參加「個人項目」；又其中任何四位可參加「團體項目」。不足四位同學的隊伍將不獲准出賽。
3. 每隊隊員必須穿著整齊校服，並由負責老師帶領，於上午9時正或以前向會場接待處註冊，同時必須出示身分證/學生證明文件，否則將被撤銷參賽資格。
4. 粵語將會被採用為指示語言。若參賽者不諳粵語，則可獲發給一份中、英文指示。比賽題目則中、英並列。
5. 每一「個人項目」包括四部份。每一隊員回答其中一部份，其他隊友不得從旁協助，否則此項目所得分數會被取消。
6. 「個人項目」中，四部份互有關連。解答第二部份之隊員需利用第一部份之答案，如此類推。
7. 每一「團體項目」亦包括四部份。但各部份不一定相關，且可由全隊共同作答。各隊員可進行討論，但必須將聲浪降至最低。
8. 比賽時，參賽者不可使用計算工具，違例者將被取消資格或扣分。
9. 參賽者如有攜帶電子通訊器材（包括平板電腦、手提電話、多媒體播放器、電子字典、具文字顯示功能的手錶、智能手錶或其他穿戴式附有通訊或資料貯存功能之科技用品）或其他響鬧裝置，應把它關掉，並放入手提包內或座位的椅下，否則大會有權取消該隊參賽資格。
10. 除另有聲明外，所有答案須為數字，並應化簡，但無需呈交證明及算草。
11. 每一項目限時三分鐘。
12. 計分辦法如下：

(甲) 準確分:	個人項目	積分	團體項目	積分
	答對第一部分	1	答對任何一部分	2
	答對第二部分	2	答對任何兩部分	4
	答對第三部分	3	答對任何三部分	7
	答對第四部分	4	答對所有四部分	10
	合共	10		

(乙) 快捷分	積分所乘倍數
參賽隊伍完成並交出答案的時間 < 1 分鐘	4
1 分鐘 ≤ 參賽隊伍完成並交出答案的時間 < 2 分鐘	3
2 分鐘 ≤ 參賽隊伍完成並交出答案的時間 < 3 分鐘	2

(丙) 獎勵分

任何一隊在某一個人/團體項目競賽中，若全部答對時，可額外獲得 20 分。

(丁) 每項目之總分

準確分×倍數 + 獎勵分

13. 如有任何疑問，參賽者須於最後一項個人/團體賽完畢後 10 分鐘內向評判團提出。所有提出之疑問，將由評判團作最後裁決。
14. 得分最高之三隊將獲得獎盃及獎品。冠軍學校可保存總冠軍盾牌至下一屆香港數學競賽。
15. 總成績將由評判團作最後裁決。

The sixth Hong Kong Mathematics Olympiad (1988/89)
Regulations (Final Event)

1. The competition consists of **10** events, which are divided into **5** individual events and **5** group events.
2. Each participating team should consist of students who have enrolled in the heat event. Any 4 of them may take part in the individual event and any 4 of them may take part in the group event. Teams of less than 4 members will not be allowed to participate.
3. Members of each team, **accompanied by the teacher-in-charge, should wear proper school uniform** and present **ID Card or student identification document** when registering at the venue reception **not later than 9:00 a.m.** Failing to do so, the team **will be disqualified**.
4. Verbal instructions will be given in Cantonese. However, for competitors who do not understand Cantonese, instructions written in both Chinese and English will be provided. Question papers are printed in both English and Chinese.
5. Each individual event consists of 4 parts. Each part must be completed by one member of the team. Help from other team members would result in disqualification for that particular event.
6. In an individual event, the four parts are interrelated. When solving Part 2, one has to make use of the answer obtained in Part 1, and so on.
7. In a group event, the four parts are to be done by the whole team and the parts may or may not be interrelated. Discussions are allowed provided that voice level is kept to a minimum.
8. Use of calculating devices will not be allowed; otherwise the team will risk disqualification or deduction of marks.
9. Participants having electronic communication devices (include tablets, mobile phones, multimedia players, electronic dictionaries, databank watches, smart watches or other wearable technologies with communication or data storage functions) or any alarm device(s), should turned them off (including the alarm function) and be put inside the bags or under the chairs. Failing to do so, the team **will risk disqualification**.
10. All answers should be numerical and reduced to the simplest form unless stated otherwise. No proof or working is required.
11. The time limit for each event is 3 minutes.
12. The Marking System is as follows:
 - (a) Scores for accuracy:

<u>Individual Events</u>	<u>Scores</u>	<u>Group Events</u>	<u>Scores</u>
Part 1 correct ...	1	Any 1 part correct	...2
Part 2 correct ...	2	Any 2 parts correct	...4
Part 3 correct ...	3	Any 3 parts correct	...7
Part 4 correct ...	4	All 4 parts correct	...10
Total	10		
 - (b) Multiplying factors for speed:

<i>Time taken for the teams to hand in their answer < 1 min.</i>	<i>4</i>
<i>1 min. ≤ Time taken for the teams to hand in their answer < 2 min.</i>	<i>3</i>
<i>2 min. ≤ Time taken for the teams to hand in their answer < 3 min.</i>	<i>2</i>
 - (c) Bonus Score:
Teams, which hand in their answers of anyone individual/group event have all the answers in that event correct, will be awarded a bonus score of 20 marks.
 - (d) Total score for each event:

$$(\text{Score for accuracy}) \times (\text{Multiplying factor}) + (\text{Bonus score})$$
13. Any queries should reach the Judging Panel within 10 minutes after the end of the last individual group event. The decision of the Judging Panel on the queries is final.
14. Trophies and prizes will be given to the three schools achieving the highest scores. The champion school may keep the Champion shield until the next Hong Kong Mathematics Olympiad.
15. The decision of the Judging Panel on the overall results is final.

決賽名單：

<u>Team No.</u>	<u>Name of School</u>
1	Baptist Lui Ming Choi Secondary School
2	Carmel Secondary School
3	CCC Mong Man Wai College
4	Chan Sui Ki (La Salle) College
5	Cheung Sha Wan Government Scondary School
6	Chiu Lut Sau Memorial Secondary School
7	Choi Hung Estate Catholic Secondary School
8	Chuen Yuen College
9	Ha Kwai Chung Government Secondary Technical School
10	Heung To Middle School (Tai Hang Tung)
11	King's College
12	Kowloon Technical School
13	Kwun Tong Government Secondary School
14	Maryknoll Convent School
15	Munsang College
16	Pui Ching Middle School
17	Pui Kiu Middle School
18	Queen Elizabeth School
19	Queen's College
20	Sacred Heart Canossiann College
21	Sha Tin Government Secondary School
22	Shun Lee Catholic Secondary School
23	Sing Yin Secondary School
24	S.K.H. Lui Ming Choi Secondary School
25	S.K.H. Tsang Shiu Tim Secondary School
26	St. Francis Xavier's School (Tsuen Wan)
27	St. Joseph's Anglo-Chinese College
28	St. Joseph's College
29	St. Mary's Canossian College
30	St. Peter's Secondary College
31	Tang Shiu Kin Victoria Technical School
32	Tsuen Wan Government Secondary School
33	Tsuen Wan Public Ho Cheun Yiu Memorial College
34	T.W.G.H. Wong Fut Nam College
35	Wa Ying College
36	Wah Yan College (Hong Kong)
37	Y.C.H. Lim Por Yen Secondary School
38	Ying Wa College
39	Ying Wa Girls' College
40	Yuen Long Public Secondary School

And the winning schools in the final event 1988-1989 were:

Championship: King's College

1st Runner-up: Ha Kwai Chung Government Secondary Technical School

2nd Runner-up: Queen's College

Hong Kong Mathematics Olympiad (1988 – 1989)

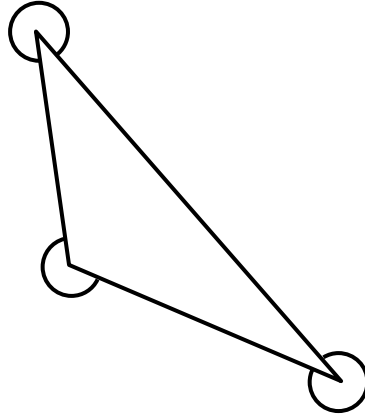
Sample Event (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 附圖所示三角的和是 a° ，求 a 的值。

In the given diagram, the sum of the three marked angles is a° . Find the value of a .



$a =$

- (ii) 一凸 b 邊形的內角和為 a° ，求 b 的值。

The sum of the interior angles of a convex b -sided polygon is a° .

Find the value of b .

$b =$

- (iii) 若 $27^{b-1} = c^{18}$ ，求 c 的值。

If $27^{b-1} = c^{18}$, find the value of c .

$c =$

- (iv) 若 $c = \log_d 125$ ，求 d 的值。

If $c = \log_d 125$, find the value of d .

$d =$

FOR OFFICIAL USE

Score for
accuracy

\times

Mult. factor for
speed

$=$

Team No.

$+$

Bonus
score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (1988 – 1989)

Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 在十時三十分，時鐘兩針構成的鈍角是 $(100 + a)^\circ$ ，求 a 的值。

The obtuse angle formed by the hands of a clock at 10:30 is $(100 + a)^\circ$.

Find the value of a .

$a =$

- (ii) 兩直綫 $ax + by = 0$ 及 $x - 5y + 1 = 0$ 互相垂直。求 b 的值。

The lines $ax + by = 0$ and $x - 5y + 1 = 0$ are perpendicular to each other.

Find the value of b .

$b =$

- (iii) 已知 $(b + 1)^4 = 2^{c+2}$ ，求 c 的值。

If $(b + 1)^4 = 2^{c+2}$, find the value of c .

$c =$

- (iv) 已知 $c - 9 = \log_c (6d - 2)$ ，求 d 的值。

If $c - 9 = \log_c (6d - 2)$, find the value of d .

$d =$

FOR OFFICIAL USE

Score for accuracy	<input type="text"/>	\times	Mult. factor for speed	<input type="text"/>	$=$	<input type="text"/>
			+	Bonus score		<input type="text"/>
				Total score		<input type="text"/>

Team No.

Time

Min.

Sec.

Hong Kong Mathematics Olympiad (1988 – 1989)

Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 已知 $1000a = 85^2 - 15^2$ ，求 a 的值。

If $1000a = 85^2 - 15^2$, find the value of a .

$a =$

- (ii) 假設點 (a, b) 在直線 $5x + 2y = 41$ 上。求 b 的值。

The point (a, b) lies on the line $5x + 2y = 41$. Find the value of b .

$b =$

- (iii) $x + b$ 是 $x^2 + 6x + c$ 的因式。求 c 的值。

$x + b$ is a factor of $x^2 + 6x + c$. Find the value of c .

$c =$

- (iv) 設 d 是兩點 $(c, 1)$ 及 $(5, 4)$ 間的距離，求 d 的值。

If d is the distance between the points $(c, 1)$ and $(5, 4)$, find the value of d .

$d =$

FOR OFFICIAL USE

Score for
accuracy

×

Mult. factor for
speed

=

Team No.

+
Bonus
score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (1988 – 1989)

Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 已知 $\alpha + \beta = 11$ ， $\alpha\beta = 24$ ，且 $\alpha > \beta$ ，求 α 的值。

If $\alpha + \beta = 11$, $\alpha\beta = 24$ and $\alpha > \beta$, find the value of α .

$\alpha =$

- (ii) 已知 $\tan \theta = \frac{-\alpha}{15}$ ， $90^\circ < \theta < 180^\circ$ ，且 $\sin \theta = \frac{b}{34}$ ，求 b 的值。

If $\tan \theta = \frac{-\alpha}{15}$, $90^\circ < \theta < 180^\circ$ and $\sin \theta = \frac{b}{34}$, find the value of b .

$b =$

- (iii) 一正方形內接一個直徑為 b 的圓。設正方形的面積為 A ，求 A 的值。

If A is the area of a square inscribed in a circle of diameter b , find the value of A .

$A =$

- (iv) 已知 $x^2 + 22x + A \equiv (x + k)^2 + d$ ，其中 k, d 是常數，求 d 的值。

If $x^2 + 22x + A \equiv (x + k)^2 + d$, where k, d are constants, find the value of d .

$d =$

FOR OFFICIAL USE

Score for
accuracy

×

Mult. factor for
speed

=

Team No.

+ Bonus
score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (1988 – 1989)

Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 已知 p, q, r 的平均數是 12，且 $p, q, r, t, 2t$ 的平均數是 15。求 t 的值。

The average of p, q, r is 12. The average of $p, q, r, t, 2t$ is 15. Find the value of t .

$t =$

- (ii) k 是實數，且 $k^4 + \frac{1}{k^4} = t + 1$ ，設 $s = k^2 + \frac{1}{k^2}$ 。求 s 的值。

k is a real number such that $k^4 + \frac{1}{k^4} = t + 1$, and $s = k^2 + \frac{1}{k^2}$. Find the value of s .

$s =$

- (iii) M 及 N 依次是 $(1, 2)$ ， $(11, 7)$ 兩點。 $P(a, b)$ 是 MN 上一點使 $MP : PN = 1 : s$ 。求 a 的值。

M and N are the points $(1, 2)$ and $(11, 7)$ respectively. $P(a, b)$ is a point on MN such that $MP : PN = 1 : s$. Find the value of a .

$a =$

- (iv) 已知曲線 $y = ax^2 + 12x + c$ 與 x -軸相切，求 c 的值。

If the curve $y = ax^2 + 12x + c$ touches the x -axis, find the value of c .

$c =$

FOR OFFICIAL USE

Score for
accuracy

×

Mult. factor for
speed

=

Team No.

+ Bonus
score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (1988 – 1989)

Final Event 5 (Individual)

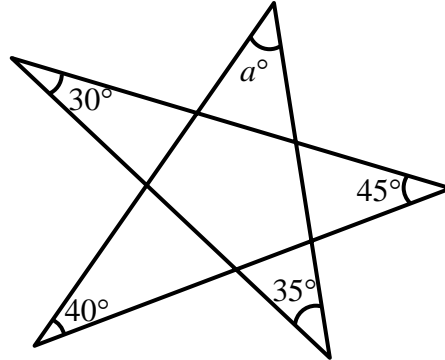
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 如圖所示，求 a 的值。

In the figure, find the value of a .

$a =$



- (ii) 已知 $\sin(a^\circ + 210^\circ) = \cos b^\circ$ ，且 $90^\circ < b < 180^\circ$ ，求 b 的值。

If $\sin(a^\circ + 210^\circ) = \cos b^\circ$, and $90^\circ < b < 180^\circ$, find the value of b .

$b =$

- (iii) 一正 n 邊形的每一內角是 b° 。求 n 的值。

Each interior angle of an n -sided regular polygon is b° . Find the value of n .

$n =$

- (iv) 某年三月第 n 日是星期五，同年三月第 k 日是星期三，且 $20 < k < 25$ 。求 k 的值。

The n^{th} day of March in a year is Friday. The k^{th} day of March in the same year is Wednesday, where $20 < k < 25$. Find the value of k .

$k =$

FOR OFFICIAL USE

Score for accuracy	<input type="text"/>	\times	Mult. factor for speed	<input type="text"/>	$=$	<input type="text"/>
			+	Bonus score		<input type="text"/>
				Total score		<input type="text"/>

Team No.

Time

Min.

Sec.

Hong Kong Mathematics Olympiad (1988 – 1989)

Sample Event (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

(i) 已知 $2at^2 + 12t + 9 = 0$ 有等根，求 a 的值。

If $2at^2 + 12t + 9 = 0$ has equal roots, find the value of a .

$a =$

(ii) 已知 $ax + by = 1$ 及 $4x + 18y = 3$ 平行，求 b 的值。

If $ax + by = 1$ and $4x + 18y = 3$ are parallel, find the value of b .

$b =$

(iii) 第 b 個質數是 p 。求 p 的值。

The b^{th} prime number is p . Find the value of p .

$p =$

(iv) 已知 $k = \frac{4\sin\theta + 3\cos\theta}{2\sin\theta - \cos\theta}$ ，且 $\tan\theta = 3$ ，求 k 的值。

If $k = \frac{4\sin\theta + 3\cos\theta}{2\sin\theta - \cos\theta}$ and $\tan\theta = 3$, find the value of k .

$k =$

FOR OFFICIAL USE

Score for
accuracy

×

Mult. factor for
speed

=

Team No.

+

Bonus
score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (1988 – 1989)

Final Event 6 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 一凸 n 邊形有 20 條對角線。求 n 的值。

An n -sided convex polygon has 20 diagonals. Find the value of n .

$n =$

- (ii) 兩骰同擲，所得點數之和是 n 的概率是 $\frac{k}{36}$ 。求 k 的值。

Two dice are thrown. The probability of getting a total of n is $\frac{k}{36}$. Find the value of k .

$k =$

- (iii) 某人以 25 km/h 的速率行車 3 小時，再以 50 km/h 的速率行車 2 小時。

若全程的平均速率是 u km/h，求 u 的值。

A man drives at 25 km/h for 3 hours and then at 50 km/h for 2 hours.

His average speed for the whole journey is u km/h. Find the value of u .

$u =$

- (iv) 已知 $a\Delta b = ab + 1$ ，且 $(2\Delta a)\Delta 3 = 10$ ，求 a 的值。

If $a\Delta b = ab + 1$ and $(2\Delta a)\Delta 3 = 10$, find the value of a .

$a =$

FOR OFFICIAL USE

Score for
accuracy

×

Mult. factor for
speed

=

Team No.

+

Bonus
score

Time

Total score

Min.

Sec.

Hong Kong Mathematics Olympiad (1988 – 1989)

Final Event 7 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

在下圖所示乘法中，不同字母代表由 1 至 9 的不同整數。設字母 O 及 J 依次代表 4 及 6。求

In the attached calculation, different letters represent different integers ranging from 1 to 9.

If the letters O and J represent 4 and 6 respectively, find

$$\begin{array}{r} G \ O \ L \ D \ E \ N \\ \times \qquad \qquad \qquad J \\ \hline D \ E \ N \ G \ O \ L \end{array}$$

- (i) G 的值。
the value of G .

- (ii) D 的值。
the value of D .

- (iii) L 的值。
the value of L .

- (iv) E 的值。
the value of E .

FOR OFFICIAL USE

Score for accuracy	<input type="text"/>	×	Mult. factor for speed	<input type="text"/>	=	<input type="text"/>
			+	Bonus score		<input type="text"/>
				Total score		<input type="text"/>

Team No.	<input type="text"/>
Time	<input type="text"/>
	<input type="text"/>
Min.	Sec.

Hong Kong Mathematics Olympiad (1988 – 1989)

Final Event 8 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 設 y 是 $\frac{14}{5+3\sin\theta}$ 的最大值。求 y 的值。

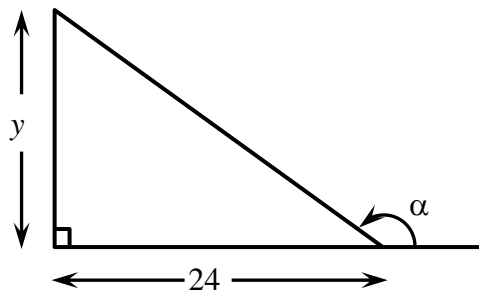
If y is the greatest value of $\frac{14}{5+3\sin\theta}$, find the value of y .

$y =$

- (ii) 如圖所示， $100 \cos \alpha = k$ 。求 k 的值。

In the figure, $100 \cos \alpha = k$. Find the value of k .

$k =$



- (iii) $3x^2 + 4x + a$ 被 $x + 2$ 除所得的餘數是 5。求 a 的值。

When $3x^2 + 4x + a$ is divided by $x + 2$, the remainder is 5. Find the value of a .

$a =$

- (iv) $3t^2 - 5t - 2 < 0$ 的解是 $-\frac{1}{3} < t < m$ 。求 m 的值。

The solution for $3t^2 - 5t - 2 < 0$ is $-\frac{1}{3} < t < m$. Find the value of m .

$m =$

FOR OFFICIAL USE

Score for
accuracy

×

Mult. factor for
speed

=

Team No.

+
Bonus
score

Time

Total score

Min.

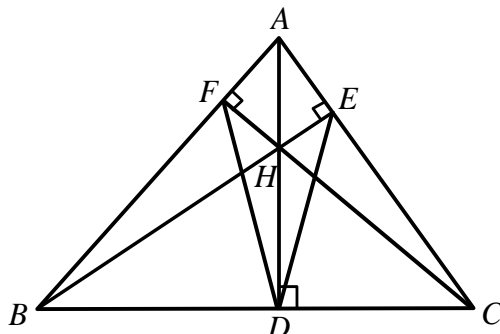
Sec.

Hong Kong Mathematics Olympiad (1988 – 1989)
Final Event 9 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
 除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 圖中， $\angle BAC = 70^\circ$ ，且 $\angle FDE = x^\circ$ ，求 x 的值。

In the figure, $\angle BAC = 70^\circ$ and $\angle FDE = x^\circ$. Find the value of x .



$x =$

- (ii) 一長方體闊 y cm，長 6 cm，高 5 cm。它的表面積是 126 cm^2 ，求 y 的值。

A cuboid is y cm wide, 6 cm long and 5 cm high. Its surface area is 126 cm^2 . Find the value of y .

$y =$

- (iii) 已知 $\log_9(\log_2 k) = \frac{1}{2}$ ，求 k 的值。

If $\log_9(\log_2 k) = \frac{1}{2}$, find the value of k .

$k =$

- (iv) 已知 $a : b = 3 : 8$ ， $b : c = 5 : 6$ ，且 $a : c = r : 16$ ，求 r 的值。

If $a : b = 3 : 8$, $b : c = 5 : 6$ and $a : c = r : 16$, find the value of r .

$r =$

FOR OFFICIAL USE

Score for
accuracy

×

Mult. factor for
speed

=

Team No.

+ Bonus
score

Time

Total score

Min.

Sec.

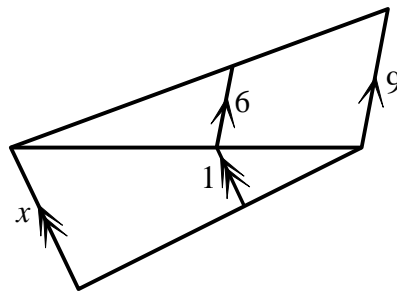
Hong Kong Mathematics Olympiad (1988 – 1989)
Final Event 10 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
 除非特別聲明，答案須用數字表達，並化至最簡。

(i) 已知 $\frac{6\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = 3\sqrt{a} + 6$ ，求 a 的值。

If $\frac{6\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = 3\sqrt{a} + 6$, find the value of a .

(ii) 如圖所示，求 x 的值。
 In the figure, find the value of x .



(iii) 已知 $k = \frac{6\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta}{\cos^2\theta + \sin\theta\cos\theta + \sin^2\theta}$ ，且 $\tan\theta = 2$ ，求 k 的值。

If $k = \frac{6\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta}{\cos^2\theta + \sin\theta\cos\theta + \sin^2\theta}$ and $\tan\theta = 2$, find the value of k .

(iv) 已知 $y = \frac{3(2^k) - 4(2^{k-2})}{2^k - 2^{k-1}}$ ，求 y 的值。

If $y = \frac{3(2^k) - 4(2^{k-2})}{2^k - 2^{k-1}}$, find the value of y .

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=	
			+	Bonus score		
				Total score		

Team No.	
Time	
Min.	Sec.