Hong Kong Mathematics Olympiad (1989 – 1990)
Sample Event (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若方程 $3x^2 - 4x + \frac{h}{3} = 0$ 有等根，求 h 的值。

If the equation $3x^2 - 4x + \frac{h}{3} = 0$ has equal roots, find the value of h.

$h =$

(ii) 若一圓柱體之高增加一倍，且新半徑為原來之 h 倍，則新體積為原來之 k 倍，求 k 的值。

If the height of a cylinder is doubled and the new radius is h times the original, then the new volume is k times the original. Find the value of k.

$k =$

(iii) 若 $\log_{10} 210 + \log_{10} k - \log_{10} 56 + \log_{10} 40 - \log_{10} 120 + \log_{10} 25 = p$, 求 p 的值。

If $\log_{10} 210 + \log_{10} k - \log_{10} 56 + \log_{10} 40 - \log_{10} 120 + \log_{10} 25 = p$, find the value of p.

$p =$

(iv) 若 $\sin A = \frac{p}{5}$ 且 $\frac{\cos A}{\tan A} = \frac{q}{15}$，求 q 的值。

If $\sin A = \frac{p}{5}$ and $\frac{\cos A}{\tan A} = \frac{q}{15}$, find the value of q.

$q =$

FOR OFFICIAL USE

Score for accuracy		× Mult. factor for speed		=		Team No.	
			+ Bonus score			Time	
			Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1989 – 1990)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $2t+1$ 是 $4t^2+12t+a$ 的因式，求 a 的值。
Find the value of a if $2t+1$ is a factor of $4t^2+12t+a$.

$a =$

(ii) 對 $K \geq 0$，$\sqrt{K}$ 表 K 的非負平方根。若 b 是方程 $\sqrt{a-x} = x-3$ 的根，求 b 的值。
$\sqrt{K}$ denotes the nonnegative square root of K, where $K \geq 0$.
If b is the root of the equation $\sqrt{a-x} = x-3$, find the value of b.

$b =$

(iii) 若 c 是 $\dfrac{20}{4+2\cos\theta}$ 的最大值，求 c 的值。
If c is the greatest value of $\dfrac{20}{4+2\cos\theta}$, find the value of c.

$c =$

(iv) 某人以 $3c$ km/h 的速率行車 3 小時，再以 $4c$ km/h 的速率行車 2 小時。若全程的平均速率是 d km/h，求 d 的值。
A man drives a car at $3c$ km/h for 3 hours and then $4c$ km/h for 2 hours.
If his average speed for the whole journey is d km/h, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]
Min. Sec.

Hong Kong Mathematics Olympiad (1989 – 1990)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $0° \le \theta < 360°$，θ 的方程 $3\cos\theta + \dfrac{1}{\cos\theta} = 4$ 有 p 個根，求 p 的值。

If $0° \le \theta < 360°$, the equation in θ: $3\cos\theta + \dfrac{1}{\cos\theta} = 4$ has p roots.

Find the value of p.

$p =$

(ii) 若 $x - \dfrac{1}{x} = p$，且 $x^3 - \dfrac{1}{x^3} = q$，求 q 的值。

If $x - \dfrac{1}{x} = p$ and $x^3 - \dfrac{1}{x^3} = q$, find the value of q.

$q =$

(iii) 一圓內接於一周界長 q cm 的正三角形。若圓的面積是 $k\pi$ cm^2，求 k 的值。

A circle is inscribed in an equilateral triangle of perimeter q cm.

If the area of the circle is $k\pi$ cm^2, find the value of k.

$k =$

(iv) 正 k 邊形的每一內角為 $m°$。求 m 的值。

Each interior angle of a regular polygon of k sides is $m°$. Find the value of m.

$m =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
					+ Bonus score		Time		
					Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1989 – 1990)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $998a + 1 = 999^2$，求 a 的值。
If $998a + 1 = 999^2$, find the value of a.

$a =$

(ii) 若 $\log_{10} a = \log_2 b$，求 b 的值。
If $\log_{10} a = \log_2 b$, find the value of b.

$b =$

(iii) 以 x 軸，y 軸及直綫 $2x + y = b$ 所圍成的三角形的面積是 c 平方單位，求 c 的值。
The area of the triangle formed by the x-axis, the y-axis and the line $2x + y = b$ is c sq. units. Find the value of c.

$c =$

(iv) 若 $64t^2 + ct + d$ 是完全平方，求 d 的值。
If $64t^2 + ct + d$ is a perfect square, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
					+ Bonus score		Time		
					Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1989 – 1990)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 解下列 a 的方程 $2^{a+1} + 2^a + 2^{a-1} = 112$。
Solve for a in the equation $2^{a+1} + 2^a + 2^{a-1} = 112$.

$a =$

(ii) 若 a 是方程 $x^2 - bx + 35 = 0$ 的一個根，求 b 的值。
If a is one root of the equation $x^2 - bx + 35 = 0$, find the value of b.

$b =$

(iii) 若 $\sin\theta = \frac{-b}{15}$，其中 $180° < \theta < 270°$，且 $\tan\theta = \frac{c}{3}$，求 c 的值。
If $\sin\theta = \frac{-b}{15}$, where $180° < \theta < 270°$, and $\tan\theta = \frac{c}{3}$, find the value of c.

$c =$

(iv) 兩骰同擲，所得點數之和為 c 的概率是 $\frac{1}{d}$。求 d 的值。
The probability of getting a sum of c in throwing two dice is $\frac{1}{d}$. Find the value of d .

$d =$

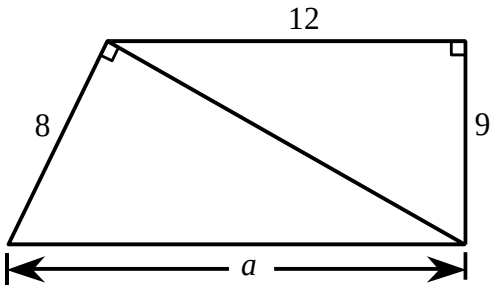
FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
				+ Bonus score			Time		
				Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1989 – 1990)
Final Event 5 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 如圖所示，求 a 的值。
In the figure, find the value of a.



$a =$

(ii) 若直綫 $ax + by = 1$ 及 $10x - 34y = 3$ 互相垂直，求 b 的值。
If the lines $ax + by = 1$ and $10x - 34y = 3$ are perpendicular to each other, find the value of b.

$b =$

(iii) 某年五月第 b 日為星期五，而同年五月第 c 日為星期二，且 $16 < c < 24$，求 c 的值。
If the b^{th} day of May in a year is Friday and the c^{th} day of May in the same year is Tuesday, where $16 < c < 24$, find the value of c.

$c =$

(iv) c 是第 d 個質數。求 d 的值。
c is the d^{th} prime number. Find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1989 – 1990)
Sample Event (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 某兩數之和為 50，其積為 25。若該兩數倒數之和為 a，求 a 的值。
The sum of two numbers is 50, and their product is 25.
If the sum of their reciprocals is a, find the value of a.

$a =$

(ii) 若直綫 $ax + 2y + 1 = 0$ 及 $3x + by + 5 = 0$ 互相垂直，求 b 的值。
If the lines $ax + 2y + 1 = 0$ and $3x + by + 5 = 0$ are perpendicular,
find the value of b.

$b =$

(iii) 一正三角形之面積為 $100\sqrt{3}\,\text{cm}^2$。若其周界為 p cm，求 p 的值。
The area of an equilateral triangle is $100\sqrt{3}\,\text{cm}^2$.
If its perimeter is p cm, find the value of p.

$p =$

(iv) 若 $x^3 - 2x^2 + px + q$ 可被 $x + 2$ 整除，求 q 的值。
If $x^3 - 2x^2 + px + q$ is divisible by $x + 2$, find the value of q.

$q =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1989 – 1990)
Final Event 6 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $a=\dfrac{\left(68^3-65^3\right)\cdot\left(32^3+18^3\right)}{\left(32^2-32\times18+18^2\right)\cdot\left(68^2+68\times65+65^2\right)}$，求 a 的值。

If $a=\dfrac{\left(68^3-65^3\right)\cdot\left(32^3+18^3\right)}{\left(32^2-32\times18+18^2\right)\cdot\left(68^2+68\times65+65^2\right)}$, find the value of a.

$a=$

(ii) 若三點(a, b)，$(10, -4)$及$(20, -3)$共綫，求 b 的值。

If the 3 points (a, b), $(10, -4)$ and $(20, -3)$ are collinear, find the value of b.

$b=$

(iii) 若在四時十五分，時鐘兩針之間的銳角是 $k°$，求 k 的值。

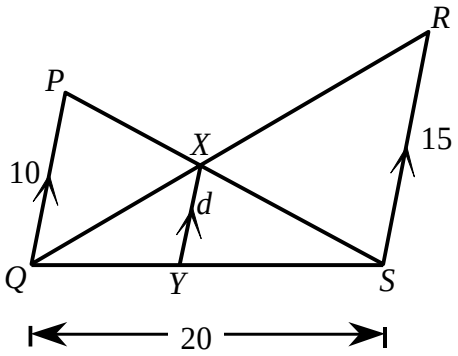
If the acute angle formed by the hands of a clock at 4:15 is $k°$, find the value of k.

$k=$

(iv) 在圖中，$PQ = 10$, $RS = 15$, $QS = 20$。若 $XY = d$，求 d 的值。

In the figure, $PQ = 10$, $RS = 15$, $QS = 20$. If $XY = d$, find the value of d.

$d=$



FOR OFFICIAL USE

Score for accuracy	×	Mult. factor for speed	=		Team No.	
			+	Bonus score	Time	
				Total score	Min.	Sec.

Hong Kong Mathematics Olympiad (1989 – 1990)
Final Event 7 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 2 個蘋果和 3 個橙共值 6 元。
4 個蘋果和 7 個橙共值 13 元。
16 個蘋果和 23 個橙共值 C 元，求 C 的值。
2 apples and 3 oranges cost 6 dollars.
4 apples and 7 oranges cost 13 dollars.
16 apples and 23 oranges cost C dollars. Find the value of C.

$C =$

(ii) 若 $K = \dfrac{6\cos\theta + 5\sin\theta}{2\cos\theta + 3\sin\theta}$，且 $\tan\theta = 2$，求 K 的值。
If $K = \dfrac{6\cos\theta + 5\sin\theta}{2\cos\theta + 3\sin\theta}$ and $\tan\theta = 2$, find the value of K.

$K =$

A、B 均為小於 10 的正整數，且 $21A104 \times 11 = 2B8016 \times 9$。
A, B are positive integers less than 10 such that $21A104 \times 11 = 2B8016 \times 9$.

(iii) 求 A 的值。
Find the value of A.

$A =$

(iv) 求 B 的值。
Find the value of B.

$B =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
					+ Bonus score		Time		
					Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1989 – 1990)
Final Event 8 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

在所示乘法中，字母 A、B、C 及 K (其中 $A < B$)代表由 1 至 9 的不同整數。

In the multiplication shown, the letters A, B, C and K ($A<B$) represent different integers from 1 to 9.

$$\begin{array}{rcc} & A & C \\ \times) & B & C \\ \hline K & K & K \end{array}$$

(i) 求 A 的值。
Find the value of A .

$A =$

(ii) 求 B 的值。
Find the value of B .

$B =$

(iii) 求 C 的值。
Find the value of C .

$C =$

(iv) 求 K 的值。
Find the value of K .

$K =$

(提示：$KKK = K\times 111$。)
(Hint: $KKK = K\times 111$.)

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
				+ Bonus score			Time		
				Total score				Min.	Sec.

Hong Kong Mathematics Olympiad (1989 – 1990)
Final Event 9 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $S = ab - 1 + a - b$，且 $a = 101$，$b = 9$，求 S 的值。
If $S = ab - 1 + a - b$ and $a = 101$, $b = 9$, find the value of S.

$S =$

(ii) 若 $x = 1.98\dot{9}$，且 $x - 1 = \dfrac{K}{99}$，求 K 的值。
If $x = 1.98\dot{9}$ and $x - 1 = \dfrac{K}{99}$, find the value of K.

$K =$

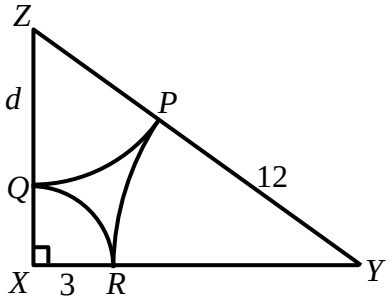
(iii) p、q 及 r 的平均值是 18。$p + 1$、$q - 2$、$r + 3$ 及 t 的平均值是 19。求 t 的值。
The average of p, q and r is 18.
The average of $p + 1$, $q - 2$, $r + 3$ and t is 19. Find the value of t.

$t =$

(iv) 如圖所示，依次以 X，Y，Z 為圓心之三弧 $\overset{\frown}{QR}$、$\overset{\frown}{RP}$、$\overset{\frown}{PQ}$ 互相切於 P、Q、R。
若 $ZQ = d$，$XR = 3$，$YP = 12$，$\angle X = 90°$，求 d 的值。

$d =$

In the figure, $\overset{\frown}{QR}$, $\overset{\frown}{RP}$, $\overset{\frown}{PQ}$ are 3 arcs, centres at X, Y and Z respectively, touching one another at P, Q and R. If $ZQ = d$, $XR = 3$, $YP = 12$, $\angle X = 90°$, find the value of d.



FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

Team No. [ ]

+ Bonus score [ ]

Time [ ] [ ]

Total score [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1989 – 1990)
Final Event 10 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $A = 1 + 2 – 3 + 4 + 5 – 6 + 7 + 8 – 9 + \ldots + 97 + 98 – 99$，求 A 的值。
If $A = 1 + 2 – 3 + 4 + 5 – 6 + 7 + 8 – 9 + \ldots + 97 + 98 – 99$, find the value of A.

$A =$

(ii) 若 $\log_{10}(k – 1) – \log_{10}(k^2 – 5k + 4) + 1 = 0$，求 k 的值。
If $\log_{10}(k – 1) – \log_{10}(k^2 – 5k + 4) + 1 = 0$, find the value of k.

$k =$

一凸 n 邊形其中一內角為 $x°$，而其餘內角之和為 2180°。
One interior angle of a convex n-sided polygon is $x°$.
The sum of the remaining interior angles is 2180°.

(iii) 求 x 的值。
Find the value of x.

$x =$

(iv) 求 n 的值。
Find the value of n.

$n =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
					+	Bonus score		Time		
						Total score			Min.	Sec.