Hong Kong Mathematics Olympiad (1990 – 1991)
Sample Event (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $a = -1 + 2 - 3 + 4 - 5 + 6 - \ldots + 100$，求 a 的值。
If $a = -1 + 2 - 3 + 4 - 5 + 6 - \ldots + 100$, find the value of a.

$a =$

(ii) 首 b 個正奇數之和是 $2a$。求 b 的值。
The sum of the first b positive odd numbers is $2a$. Find the value of b.

$b =$

(iii) 袋中有白球 b 個，黑球 3 個。現任意取出二球。
若得到兩個不同顏色的球的概率為 $\frac{c}{13}$，求 c 的值。
A bag contains b white balls and 3 black balls. Two balls are drawn from the bag at random. If the probability of getting 2 balls of different colours is $\frac{c}{13}$, find the value of c.

$c =$

(iv) 若直綫 $cx + 10y = 4$ 及 $dx - y = 5$ 互相垂直，求 d 的值。
If the lines $cx + 10y = 4$ and $dx - y = 5$ are perpendicular to each other, find the value of d.

$d =$

FOR OFFICIAL USE

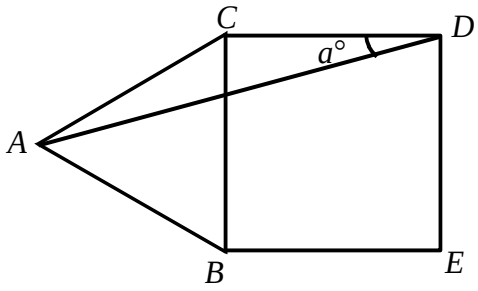
Score for accuracy		× Mult. factor for speed		=		Team No.		
				+ Bonus score		Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1990 – 1991)
Final Event 1 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 如圖所示，ABC 是等邊三角形，$BCDE$ 是正方形。若 $\angle ADC = a°$，求 a 的值。
In the figure, ABC is an equilateral triangle and $BCDE$ is a square.
If $\angle ADC = a°$, find the value of a.

$a =$



(ii) 若 $rb = 15$，且 $br^4 = 125a$，其中 r 是整數，求 b 的值。
If $rb = 15$ and $br^4 = 125a$, where r is an integer, find the value of b.

$b =$

(iii) 若方程 $bx^2 - 252x - 13431 = 0$ 之正根是 c，求 c 的值。
If the positive root of the equation $bx^2 - 252x - 13431 = 0$ is c, find the value of c.

$c =$

(iv) 已知 $x \# y = \dfrac{y-1}{x} - x + y$。若 $d = 10 \# c$，求 d 的值。
Given $x \# y = \dfrac{y-1}{x} - x + y$. If $d = 10 \# c$, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
					+	Bonus score		Time		
						Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1990 – 1991)
Final Event 2 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $a^2 - 1 = 123 \times 125$，且 $a > 0$，求 a 的值。
If $a^2 - 1 = 123 \times 125$ and $a > 0$, find the value of a.

$a =$

(ii) 若 $x^3 - 16x^2 - 9x + a$ 除以 $x - 2$ 之餘數為 b，求 b 的值。
If the remainder of $x^3 - 16x^2 - 9x + a$ when divided by $x - 2$ is b, find the value of b.

$b =$

(iii) 若一凸 n 邊形有$(b + 4)$條對角綫，求 n 的值。
If an n-sided polygon has $(b + 4)$ diagonals, find the value of n.

$n =$

(iv) 若點$(3, n)$、$(5, 1)$、$(7, d)$共綫，求 d 的值。
If the points $(3, n)$, $(5, 1)$ and $(7, d)$ are collinear, find the value of d.

$d =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
					+	Bonus score		Time		
						Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1990 – 1991)
Final Event 3 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 6 位數 168a26 可被 3 整除，求 a 之最大可能值。
If the 6-digit number 168a26 is divisible by 3,
find the greatest possible value of a.

$a =$

(ii) 一個邊長 a cm 之正方體在全部面上都塗上紅色後，再被分割為邊長 1 cm 之正方體。若所有面都未有被塗上顏色之正方體數目為 b，求 b 的值。
A cube with edge a cm long is painted red on all faces.
It is then cut into cubes with edge 1 cm long.
If the number of cubes with all the faces not painted is b, find the value of b.

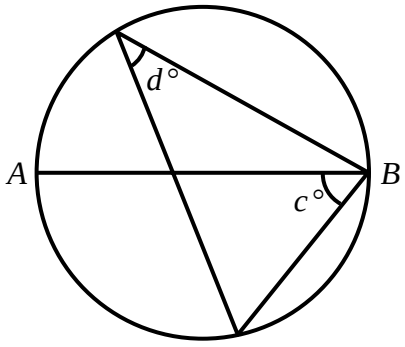
$b =$

(iii) 若 $(x-85)(x-c) \equiv x^2 - bx + 85c$，求 c 的值。
If $(x-85)(x-c) \equiv x^2 - bx + 85c$, find the value of c.

$c =$

(iv) 在圖中，AB 是該圓形的直徑。求 d 的值。
In the figure, AB is a diameter of the circle. Find the value of d.

$d =$



FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
					+ Bonus score		Time		
					Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1990 – 1991)
Final Event 4 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) Given $x-\frac{1}{x}=3$. If $a=x^2+\frac{1}{x^2}$, find the value of a.

已知 $x-\frac{1}{x}=3$。若 $a=x^2+\frac{1}{x^2}$，求 a 的值。

$a=$

(ii) 若 $f(x)=\log_2 x$，且 $f(a+21)=b$，求 b。

If $f(x)=\log_2 x$ and $f(a+21)=b$, find b.

$b=$

(iii) 若 $\cos\theta=\frac{8b}{41}$，其中 θ 為銳角，且 $c=\frac{1}{\sin\theta}+\frac{1}{\tan\theta}$，求 c 的值。

If $\cos\theta=\frac{8b}{41}$, where θ is an acute angle, and $c=\frac{1}{\sin\theta}+\frac{1}{\tan\theta}$, find the value of c.

$c=$

(iv) 兩骰同擲，得和為 7 或 c 之概率為 $\frac{d}{18}$，求 d 的值。

Two dice are tossed. If the probability of getting a sum of 7 or c is $\frac{d}{18}$, find the value of d.

$d=$

FOR OFFICIAL USE

Score for accuracy		× Mult. factor for speed		=		Team No.		
				+ Bonus score		Time		
				Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1990 – 1991)
Final Event 5 (Individual)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 在圖一中，若多邊形之內角和是 $a°$，求 a 的值。
In Figure 1, if the sum of the interior angles is $a°$, find the value of a.

$a =$

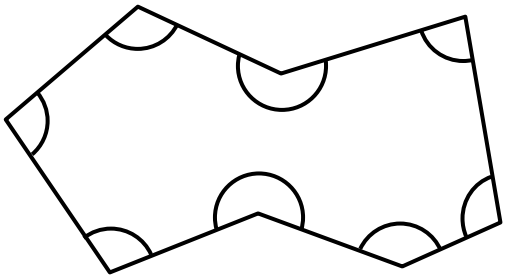


Figure 1 (圖一)

(ii) 若算術級數 80, 130, 180, 230, 280, … 之第 n 項是 a，求 n 的值。
If the n^{th} term of the arithmetic progression 80, 130, 180, 230, 280, ⋯ is a, find the value of n.

$n =$

(iii) 在圖二中，$AP : PB = 2 : 1$。若 $AC = 33$ cm，$BD = n$ cm，$PQ = x$ cm，求 x 的值。
In Figure 2, $AP : PB = 2 : 1$. If $AC = 33$ cm, $BD = n$ cm, $PQ = x$ cm, find the value of x.

$x =$

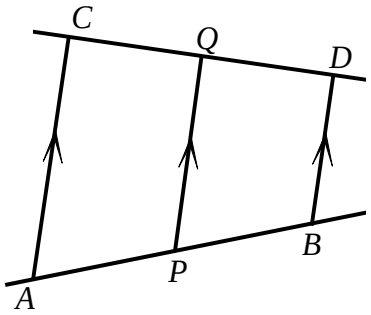


Figure 2 (圖二)

(iv) 若 $K = \dfrac{\sin 65^\circ \tan^2 60^\circ}{\tan 30^\circ \cos 30^\circ \cos x^\circ}$，求 K 的值。
If $K = \dfrac{\sin 65^\circ \tan^2 60^\circ}{\tan 30^\circ \cos 30^\circ \cos x^\circ}$, find the value of K.

$K =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
					+	Bonus score		Time		
						Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1990 – 1991)
Sample Event (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 一等邊三角形的高是 $8\sqrt{3}$ cm，面積是 $a\sqrt{3}$ cm^2。求 a 的值。
The height of an equilateral triangle is $8\sqrt{3}$ cm and the area of the triangle is $a\sqrt{3}$ cm^2. Find the value of a.

$a =$

(ii) 已知 $\sum_{x=1}^{n}\frac{1}{x}=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}$，及 $\sum_{x=4}^{10}\frac{1}{x-2}-\sum_{x=4}^{10}\frac{1}{x-1}=\frac{b}{18}$。求 b 的值。

Given that $\sum_{x=1}^{n}\frac{1}{x}=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}$, and $\sum_{x=4}^{10}\frac{1}{x-2}-\sum_{x=4}^{10}\frac{1}{x-1}=\frac{b}{18}$. Find the value of b.

$b =$

某童把一平行四邊形兩鄰邊相乘當作該圖形之面積，其結果為正確答案之兩倍。
若該圖形之銳角及鈍角依次為 $h°$及 $k°$。
A boy tries to find the area of a parallelogram by multiplying together the lengths of two adjacent sides. His answer is double the correct answer.
If the acute angle and the obtuse angle of the figure are $h°$ and $k°$ respectively,

(iii) 求 h 的值。
find the value of h.

$h =$

(iv) 求 k 的值。
find the value of k.

$k =$

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
					+	Bonus score		Time		
						Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1990 – 1991)
Final Event 6 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

某兩位數 x 之個位數字是 M，十位數字是 N。另一兩位數 y 之個位數字是 N，十位數字是 M。若 $x>y$，且他們的和是他們的差的十一倍，

A 2-digit number x has M as the units digit and N as the tens digit. Another 2-digit number y has N as the units digit and M as the tens digit.

If $x > y$ and their sum is equal to eleven times their differences,

(i) 求 M 的值。
find the value of M.

$M =$

(ii) 求 N 的值。
find the value of N.

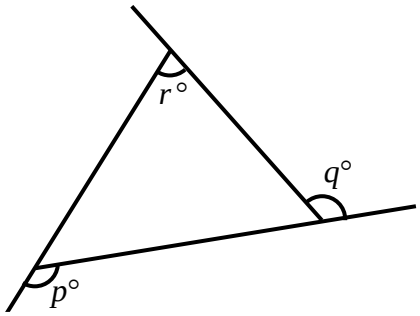
$N =$

(iii) 兩數之和是 20，積是 5。若該兩數倒數之和是 z，求 z 的值。
The sum of two numbers is 20 and their product is 5.
If the sum of their reciprocals is z, find the value of z.

$z =$

(iv) 圖中，p 與 q 的平均值是 $121 + z$。求 r 的值。
In the figure, the average of p and q is $121 + z$. Find the value of r.

$r =$



FOR OFFICIAL USE

Score for accuracy	×	Mult. factor for speed	=		Team No.	
			+	Bonus score	Time	
				Total score	Min.	Sec.

Hong Kong Mathematics Olympiad (1990 – 1991)
Final Event 7 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 5 部印刷機可在 5 天內印 5 本書。
若要在 100 天內印 100 本書，則需要 n 部印刷機，求 n 的值。
5 printing machines can print 5 books in 5 days. If n printing machines are required in order to have 100 books printed in 100 days, find the value of n.

$n =$

(ii) 某方程 $x^2 + 2x + c = 0$ 無實根，且 c 為小於 3 之整數，求 c 的值。
If the equation $x^2+2x+c = 0$ has no real root and c is an integer less than 3, find the value of c.

$c =$

雞蛋每只\$0.50，鴨蛋每只\$0.60，鵝蛋每只\$0.90。某人賣出 x 只雞蛋，y 只鴨蛋，z 只鵝蛋，共得\$60。
若 x、y、z 皆為正數，且 $x + y + z = 100$，及在 x、y、z 中有兩數相同，
Chicken eggs cost \$0.50 each, duck eggs cost \$0.60 each and goose eggs cost \$0.90 each.
A man sold x chicken eggs, y duck eggs, z goose eggs and received \$60.
If x, y, z are all positive numbers with $x + y + z = 100$ and two of the values x, y, z are equal,

(iii) 求 x 的值。
find the value of x.

$x =$

(iv) 求 y 的值。
find the value of y.

$y =$

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

Team No. [ ]

\+ Bonus score [ ]

Time [ ] [ ]

Total score [ ]

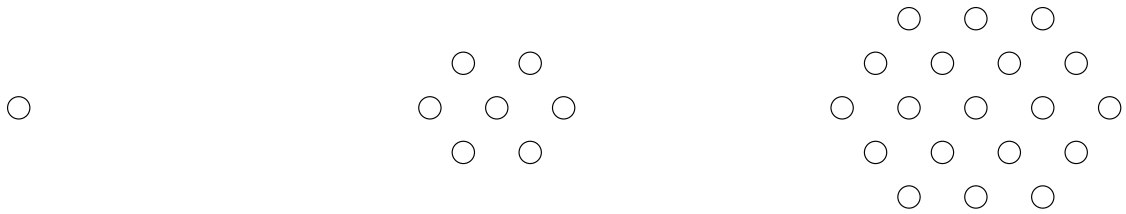
Min. Sec.

Hong Kong Mathematics Olympiad (1990 – 1991)
Final Event 8 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

細看以下之六邊形數：

Consider the following hexagonal numbers :



$H_1 = 1$ $H_2 = 7$ $H_3 = 19$

(i) 求 H_5 的值。

Find the value of H_5 .

$H_5 =$

(ii) 若 $H_n = an^2 + bn + c$，其中 n 為正整數，求 a 的值。

If $H_n = an^2 + bn + c$, where n is any positive integer, find the value of a .

$a =$

(iii) 若 $p : q = 2 : 3$，$q : r = 4 : 5$，且 $p : q : r = 8 : t : 15$，求 t 的值。

If $p : q = 2 : 3$, $q : r = 4 : 5$ and $p : q : r = 8 : t : 15$, find the value of t.

$t =$

(iv) 若 $\frac{1}{x} : \frac{1}{y} = 4 : 3$，且 $\frac{1}{x+y} : \frac{1}{x} = 3 : m$，求 m 的值。

If $\frac{1}{x} : \frac{1}{y} = 4 : 3$ and $\frac{1}{x+y} : \frac{1}{x} = 3 : m$, find the value of m .

$m =$

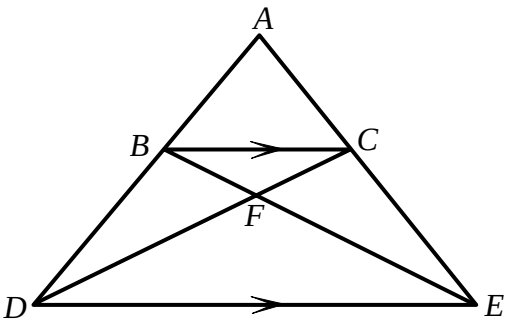
FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
					+	Bonus score		Time		
						Total score			Min.	Sec.

Hong Kong Mathematics Olympiad (1990 – 1991)
Final Event 9 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

圖中，BC 與 DE 平行。
若 $AB : BC : BF : CF : FE = 5 : 4 : 2 : 3 : 5$，
且ΔBCF 之面積為 12，求
In the figure, BC is parallel to DE.
If $AB : BC : BF : CF : FE = 5 : 4 : 2 : 3 : 5$
and the area of ΔBCF is 12, find



(i) ΔBDF 之面積，
the area of ΔBDF,

Area of ΔBDF =

(ii) ΔFDE 之面積，
the area of ΔFDE,

Area of ΔFDE =

(iii) ΔABC 之面積。
the area of ΔABC.

Area of ΔABC =

(iv) 若一球體之體積增加 72.8%，則其表面面積增加 x%。求 x 的值。
If the volume of a sphere is increased by 72.8%, then the surface area of the sphere is increased by x%. Find the value of x.

x =

FOR OFFICIAL USE

Score for accuracy [ ] × Mult. factor for speed [ ] = [ ]

\+ Bonus score [ ]

Total score [ ]

Team No. [ ]

Time [ ] [ ]

Min. Sec.

Hong Kong Mathematics Olympiad (1990 – 1991)
Final Event 10 (Group)

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.
除非特別聲明，答案須用數字表達，並化至最簡。

在所附除法算式中
In the attached division

```
                1  D  E
        ┌──────────────
2  1  5 ) A  7  B  9  C
          F  G  H
        ──────────────
          J  5  K  9
          L  5  M  5
        ──────────────
                N  4  P
                Q  R  S
```

(i) 求 A 的值。
find the value of A.

$A =$

(ii) 求 B 的值。
find the value of B.

$B =$

(iii) 求 C 的值。
find the value of C.

$C =$

(iv) 求 D 的值。
find the value of D.

$D =$

FOR OFFICIAL USE

Score for accuracy	×	Mult. factor for speed	=		Team No.	
			+	Bonus score	Time	
				Total score	Min.	Sec.